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A Dynamic Programming Model of Energy Storage

and g n ration capacity to s rv charging oads with r ativ y high PE p n tration v s hus at ast initia y th n gativ i pacts of PE s ay b at th distribution v

h distribution syst is nor a y bui t to acco odat th anticipat d p a d and his can b in ci nt how v r sinc th syst ay on y achi v this p a during a handfu of hours ach y ar An a t rnativ is to sit storag on th constrain d sid of th distribution syst By charging storag wh n distribution is unconstrain d and discharging wh n oads ar high r th distribution syst can b downsiz d Mor ov r such storag can provid additiona va u to th uti ity syst op rator O or custo rs b yond th distribution b n ts For instanc by op rating storag in a dyna ic is anding od it can provid bac up n rgy to custo rs if th r is a s rvic outag i i ar y storag can b us d to provid anci ary s rvic s A or to arbitrag diurna day ah ad or r a ti n rgy pric di r nc s A ar xc ss g n rating capacity that a uti ity or O r s rv s to provid a bu r for r a ti d viations b tw n actua and for cast d n rgy d and or supp y Nourai 2 discuss s a M distribut d sodiu su fur batt ry us d by A rican E ctric Pow r AEP to r i v a distribution v transfor r in st irginia

sing storag for u tip app ications pr s nts op ratio

V^{L} p na ty for uns rv d bui ding oad h
γ hour y discount factor

Our od assu s that th storag has a ini u storag v R^{min} his accounts for t chno ogi s such as ithiu ion batt ri s which su r x tr cyc if d gradation if th stat of charg fa s too ow h unit ss ratios η^{c} and η^{d} r ct ci ncy oss s fro charging and discharging stor ag r sp ctiv y

P^{tr} is th transfor r s rat d pow r capacity h transfor r can b op rat d abov this capacity how v r which acc rat s transfor r aging and i pos s a cost his acc rat d aging is typica y sti at d using hot spot or top oi t p ratur od s usa t a p and Gong t a pr s nt two xa p s of such od s h s od s sti at th ct of op rating th transfor r abov its rat d capacity on its ov ra if ti Co bining this aging ct with an assu d transfor r r p ac nt cost giv s a cost for op rating th transfor r abov its rat d capacity which w d not $V^{tr}(v$ his function which w assu to b conv x accru s on an hour y basis and r pr s nts th cost incurr d in ach hour during which th transfor r is op rat d abov its rat d capacity A owing th transfor r to b op rat d abov its rat d capacity is an add d f atur of our od co par d to that d v op d by Xi t a V^{L} is th cost p na ty for curtai ing distribution v oads which ar caus d by syst outag s and distribution constraints tor d n rgy can b us d how v r to r duc such curtai nts

2 2 D cision ariab s

e_t^{d} n rgy discharg d for sa s in hour t h
e_t^{c} n rgy charg d into storag in hour t h
e_t^{l} n rgy discharg d fro storag in hour t to s rv distribution v oad h
l_t distribution v oad t in hour t h
k_t r gu ation capacity so d in hour t h
v_t a ount transfor r is ov r oad d in hour t –

a so d n A_t $(e_t^{d}, e_t^{c}, e_t^{l}, l_t, k_t, v_t)$ as a v ctor of hour t d cision vari ab s

2 tat ariab s

x_t tota n rgy in storag at th b ginning of hour t h
p_t^{e} ar t pric of n rgy in hour t h
p_t^{r} ar t pric of r gu ation in hour t h
D_t distribution v n rgy d and in hour t h
I_t binary variab indicating if th r is a syst outag qua s in hour t
δ_t^{u} dispatch to contract ratio of r gu ation up in hour t –

$$I_{t+1} \quad I_{t+1} \quad I_{t},$$

$$\delta^{u}_{t+1} \quad \delta^{u}_{t+1} \quad \delta^{u}_{t},$$

and

$$\delta^{d}_{t+1} \quad \delta^{d}_{t+1} \quad \delta^{d}_{t}.$$

Constraints (and (a so d fn $\mathbf{v_{t+1}}$ th a ount that th transfor r is op rating abov its rat d capacity

Algorithm Phase of ADP Algorithm: Estimation of $\mathbf{F}_t^{post}$

Initialize:
- Discretize $\mathbf{a}_t$, $\boldsymbol{\omega}_t$, $\mathbf{x}_t^{post}$
- Initialize $\hat{\mathbf{m}}_t(\tilde{\boldsymbol{\omega}}_t)$
- Fix $\mathbf{x}$

for $\mathbf{j} = 1$ to $\mathbf{J}$ **do**
  Randomly generate a sample path, $\{\boldsymbol{\omega}_t\}_{t}^{T}$, from the continuous distribution
  Round the continuous sample path to the nearest discretized sample path, $\{\tilde{\boldsymbol{\omega}}_t\}_t^{T}$
  for $\mathbf{t} = 1$ to $\mathbf{T} - 1$ **do**
    $(\tilde{\mathbf{a}}_t, \mathbf{y}_t) \in \arg\max \left\{ \hat{\mathbf{F}}_t^*(\mathbf{x}_t, \tilde{\boldsymbol{\omega}}_t, \hat{\mathbf{m}}_t^{j-1}(\tilde{\boldsymbol{\omega}}_t)) \middle| \tilde{\mathbf{a}}_t \in \mathcal{A}_{\mathbf{S}_t}, (19), (20) \right\}$
    Without loss of generality, suppose $\mathbf{f}^{post}(\mathbf{x}_t, \tilde{\mathbf{a}}_t)$ is the $\boldsymbol{\xi}$th discretized value of $\tilde{\mathbf{x}}_t^{post}$ in rank ordering (*i.e.*, $\mathbf{f}^{post}(\mathbf{x}_t, \tilde{\mathbf{a}}_t) = \tilde{\mathbf{x}}_{t,\xi}^{post}$)
    $\mathbf{n}_{t+1}^{u} \leftarrow \max\{0, \tilde{\boldsymbol{\delta}}_{t+1}^{u} \cdot \tilde{\mathbf{k}}_t - \boldsymbol{\eta}^{d} \cdot (\mathbf{x}_t - \bar{\mathbf{R}}^{min}) + \tilde{\mathbf{e}}_t^{d} + \tilde{\mathbf{e}}_t^{l} - \tilde{\mathbf{e}}_t^{c}\}$ and $\mathbf{n}_{t+1}^{d} \leftarrow \max\{0, \tilde{\boldsymbol{\delta}}_{t+1}^{d} \cdot \tilde{\mathbf{k}}_t - (\bar{\mathbf{R}}^{max} - \mathbf{x}_t)/\boldsymbol{\eta}^{c} - \tilde{\mathbf{e}}_t^{d} - \tilde{\mathbf{e}}_t^{l} + \tilde{\mathbf{e}}_t^{c}\}$ {compute unserved regulation}
    $\Delta\tilde{\mathbf{x}}_{t+1} \leftarrow \boldsymbol{\eta}^{c} \cdot (\tilde{\boldsymbol{\delta}}_{t+1}^{d} \cdot \tilde{\mathbf{k}}_t - \mathbf{n}_{t+1}^{d}) - (\tilde{\boldsymbol{\delta}}_{t+1}^{u} \cdot \tilde{\mathbf{k}}_t - \mathbf{n}_{t+1}^{u})/\boldsymbol{\eta}^{d}$ {compute change in storage level from regulation calls}
    $\tilde{\boldsymbol{\theta}}_{t+1,\xi} \leftarrow \left[\hat{\mathbf{F}}_{t+1}^*(\tilde{\mathbf{x}}_{t,\xi}^{post} + \Delta\tilde{\mathbf{x}}_{t+1}, \tilde{\boldsymbol{\omega}}_{t+1}, \hat{\mathbf{m}}_{t+1}^{j-1}(\tilde{\boldsymbol{\omega}}_{t+1})) - \hat{\mathbf{F}}_{t+1}^*(\tilde{\mathbf{x}}_{t,\xi-1}^{post} + \Delta\tilde{\mathbf{x}}_{t+1}, \tilde{\boldsymbol{\omega}}_{t+1}, \hat{\mathbf{m}}_{t+1}^{j-1}(\tilde{\boldsymbol{\omega}}_{t+1}))\right] / \left(\tilde{\mathbf{x}}_{t,\xi}^{post} - \tilde{\mathbf{x}}_{t,\xi-1}^{post}\right)$ and $\tilde{\boldsymbol{\theta}}_{t+1,\xi+1} \leftarrow \left[\hat{\mathbf{F}}_{t+1}^*(\tilde{\mathbf{x}}_{t,\xi+1}^{post} + \Delta\tilde{\mathbf{x}}_{t+1}, \tilde{\boldsymbol{\omega}}_{t+1}, \hat{\mathbf{m}}_{t+1}^{j-1}(\tilde{\boldsymbol{\omega}}_{t+1})) - \hat{\mathbf{F}}_{t+1}^*(\tilde{\mathbf{x}}_{t,\xi}^{post} + \Delta\tilde{\mathbf{x}}_{t+1}, \tilde{\boldsymbol{\omega}}_{t+1}, \hat{\mathbf{m}}_{t+1}^{j-1}(\tilde{\boldsymbol{\omega}}_{t+1}))\right] / \left(\tilde{\mathbf{x}}_{t,\xi+1}^{post} - \tilde{\mathbf{x}}_{t,\xi}^{post}\right)$
    for $\mathbf{i} = 1, \ldots, \mathbf{M}_t$ and $\mathbf{z} \in \tilde{\boldsymbol{\Omega}}_t$ **do**

$$\boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}) \leftarrow \begin{cases} (1-\boldsymbol{\alpha}_j) \cdot \hat{\mathbf{m}}_t^{j-1}(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}) + \boldsymbol{\alpha}_j \cdot \tilde{\boldsymbol{\theta}}_{t+1,i}, & \text{if } \mathbf{z} = \tilde{\boldsymbol{\omega}}_t \text{ and } \mathbf{i} \in \{\boldsymbol{\xi}, \boldsymbol{\xi}+1\}; \\ \hat{\mathbf{m}}_t^{j-1}(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}), & \text{otherwise} \end{cases}$$

    end for
    for $\mathbf{i} = 1, \ldots, \mathbf{M}_t$ and $\mathbf{z} \in \tilde{\boldsymbol{\Omega}}_t$ **do**

$$\hat{\mathbf{m}}_t^{j}(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}) \leftarrow \begin{cases} \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,\xi}^{post}, \tilde{\boldsymbol{\omega}}_t), & \text{if } \mathbf{z} = \tilde{\boldsymbol{\omega}}_t, \tilde{\mathbf{x}}_{t,i}^{post} < \tilde{\mathbf{x}}_{t,\xi}^{post} \text{ and } \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}) \le \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,\xi}^{post}, \tilde{\boldsymbol{\omega}}_t) \\ \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,\xi+1}^{post}, \tilde{\boldsymbol{\omega}}_t), & \text{if } \mathbf{z} = \tilde{\boldsymbol{\omega}}_t, \tilde{\mathbf{x}}_{t,i}^{post} > \tilde{\mathbf{x}}_{t,\xi+1}^{post} \text{ and } \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,i}^{post}, \mathbf{z}) \ge \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,\xi+1}^{post}, \tilde{\boldsymbol{\omega}}_t) \\ \boldsymbol{\rho}_t(\tilde{\mathbf{x}}_{t,i}^{post}, \tilde{\boldsymbol{\omega}}_t), & \text{otherwise} \end{cases}$$

    end for
    $\mathbf{x}_{t+1} \leftarrow \mathbf{f}^{post}(\mathbf{x}_t, \tilde{\mathbf{a}}_t)$

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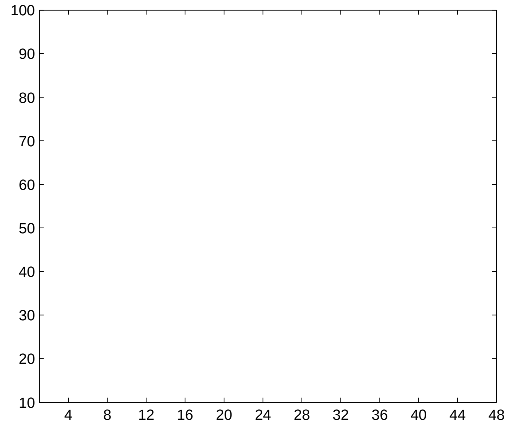
A or t Phas 2 of A DP A gorith Obtain a N ar Opti a Po icy

Fix $\mathbf{x}$
for $t = 1$ to T **do**
  Observe ω_t from continuous distribution and round it to the nearest discrete $\tilde{\omega}_t$
  if $t > 1$ **then**
    $n_t^u \leftarrow \max\{0, \delta_t^u \cdot k_{t-} - \eta^d \cdot (x_{t-} -$ $^-$

od d ini u and axi u capacity for ach w as a p rc ntag of R^{nom} Figur p furth r indicat s that th batt ri s charging and discharging ci nci s ar t p ratur d p nd nt



its rat d capacity is giv n by th fo owing conv x pi c wis in ar function

$$V^{tr}(v_t \quad \begin{cases} . \quad v_t, & \text{if } v_t \in \quad , \ . \ P^{tr} \ , \\ . \quad (v_t - \ . \ P^{tr} \quad . \quad \cdot \ . \ P^{tr}, & \text{if } v_t \in \quad . \ P^{tr}, 2 \ P^{tr} \ , \\ p. \quad (v_t - 2 \ P^{tr} \quad 2 \quad \cdot \ . \ P^{tr}, & \text{if } v_t \in \ 2 \ P^{tr}, \ . \ P^{tr} \ , \\ . \quad (v_t - \ . \ P^{tr} \quad p \quad . \ P^{tr}, & \text{if } v_t \in \quad . \ P^{tr}, \ . \ P^{tr} \ , \\ p2 \quad (v_t - \ . \ P^{tr} \quad 2 \quad . \quad \cdot \ . \ P^{tr}, & \text{if } v_t \in \quad . \ P^{tr}, \ pP^{tr} \ , \\ p \quad (v_t - \ pP^{tr} \quad . \quad \cdot \ . \ P^{tr}, & \text{if } v_t \in \ pP^{tr}, \ . \ P^{tr} \ , \\ p \quad (v_t - \ . \ P^{tr} \quad 2 \ 2 \quad \cdot \ . \ P^{tr}, & \text{if } v_t \in \quad . \ P^{tr}, \ \infty \ . \end{cases} \qquad 2$$

Exog nous Rando ariab s

A though our od and so ution a gorith do not r quir any sp ci c cor r ation a ong th xog nous rando variab s (oth r than th Mar ovian prop rty w assu in our cas study that p_t^e p_t^r D_t I_t δ_t^u and δ_t^d ar a utua y ind p nd nt furth r assu that p_t^e p_t^r D_t δ_t^u and δ_t^d ar s ria y ind p nd nt ov r ti

Historica n rgy and r gu ation capacity pric s show v ry itt corr ation In2 th s pric s had a corr ation of about $-$. p his is b caus high n rgy pric s signa ss g n rating capacity b ing avai ab and high r cost g n ration having to b us d to s rv th oad High r gu ation pric s signa a ac of fast r sponding g n ration Ind d n rgy pric s t nd to p a idding capAA gross r gu ation up and down n rgy d p oy d in r a ti h data do not show any diurna or s asona patt rns in th ratios hus w assu that th distributions ar ti invariant Hypoth sis t sting sugg sts that a Gaus sian distribution b st fits th historica data which w assu Maxi u i ihood sti ators of th an and standard d viation ar us d and (p discr tiz th $\mathbf{v_t}$ variab s using th br a points shown in qua tion 2 h distributions of th $\mathbf{p^e_t}$ δ^u_t $\mathbf{D_t}$ and δ^d_t rando variab s ar discr tiz d into v possib outco s and th distributions of th $\mathbf{p^r_t}$ rando variab s into four possib va u s using brac t dians h distribution of $\mathbf{I_t}$ n ds no discr tization as it can on y ta on two va u s

h s assu ptions yi d a discr t dyna ic progra giv n by which can b so v d using th dyna ic progra ing a gorith Mor ov r w can xp oit th structur of our prob to furth r r duc th f asib action spac ov r which w ust s arch for an opti a so ution at t p of A gorith 2 rst not that du to roundtrip ci ncy oss s it is subopti a to si u tan ous y charg and discharg n rgy hus for a t $\mathbf{e^c_t}$ cannot b non z ro if at ast on of $\mathbf{e^d_t}$ and $\mathbf{e^l_t}$ is and *vice versa* H nc th tota nu b r of co binations that $\mathbf{e^c_t}$ $\mathbf{e^d_t}$ and $\mathbf{e^l_t}$ can ta in a giv n hour is M^2 wh r

$$M \quad \left|\{R^{\mathbf{min}}, R^{\mathbf{min}} \quad p, R^{\mathbf{min}} \quad . , \cdots , R^{\mathbf{max}}\}\right| .$$

Furth r or du to th high p na ty on uns rv d bui ding oads th va u of $\mathbf{l_t}$ can b d t r in d by through and th va u s of $\mathbf{I_t}$ $\mathbf{e^c_t}$ $\mathbf{e^d_t}$ $\mathbf{e^l_t}$ $\mathbf{k_t}$ and $\mathbf{v_t}$ p ci ca y if $\mathbf{I_t}$ th n $\mathbf{l_t}$ in$\{\mathbf{D_t}, \mathbf{e^l_t}\}$ Oth rwis if $\mathbf{I_t}$ th n $\mathbf{l_t}$ in$\{\mathbf{D_t}, \mathbf{P}^{\mathbf{tr}} \quad \mathbf{v_t} \quad \mathbf{e^l_t}\}$ his structur i p i s that a axi u of $M^2 \cdot |\mathbf{v_t}| \cdot \mathbf{P}^{\mathbf{tr}}$ co binations of action variab s ar f asib and cou d b opti a in

A DP A gorith I p ntation

Nasci nto and Pow 2 prov that th pi c wis in ar approxi ations of th $\mathbf{F^{post}_t}$ functions which ar sti at d in A gorith conv rg to th batt ry co s fro r gu ation and that it provid s uch or r gu ation than arbitrag his is b caus r gu ation is pri ari y a capacity s rvic r su ting in r ativ y itt n rgy charging or discharging his ans that this s rvic t nds to incur itt cost and co parab y high r v nu s A though th dispatch to contract ratio in a particu ar hour can b high *e g* w nd cas s of up to p in th historica PJM data th r gu ation up and down signa s t nd to canc out in th ong run Our si u ation has high ratios of up to but th av rag ratio ov r th y ar is uch ow r at which is consist nt with th historica PJM data hus on av rag providing r gu ation r su ts

Table [illegible] Upper and lower bounds on optimal DP objective function value

Bound	Value [$]	Standard Error [$]
$\mathbf{B_L}$	2172.7	18.74
$\mathbf{B_U}$	2215.4	22.39

Long r Distribution Infrastructur D sign

Opti izing distribution infrastructur d sign invo v s an cono ic trad o b tw n th upfront transfor r upgrad and batt ry insta ation capita costs and th associat d str a of r v nu s and av rt d costs h s ar co bin d with an assu d annua discount rat to co put th n t pr s nt va u NP of di r nt infrastructur d signs and to d t r in an NP axi izing d p oy nt co put th s NP s ov r a y ar p riod which is a standard d sign if of a distribution v transfor r B for pr s nting th r su ts of this ana ysis w d tai th cost and r v nu assu ptions und r ying it

5.2. Cost Assumptions

Our cost assu ptions ar bas d on typica va u s r port d to us by AEP h transfor r is assu dvsa r

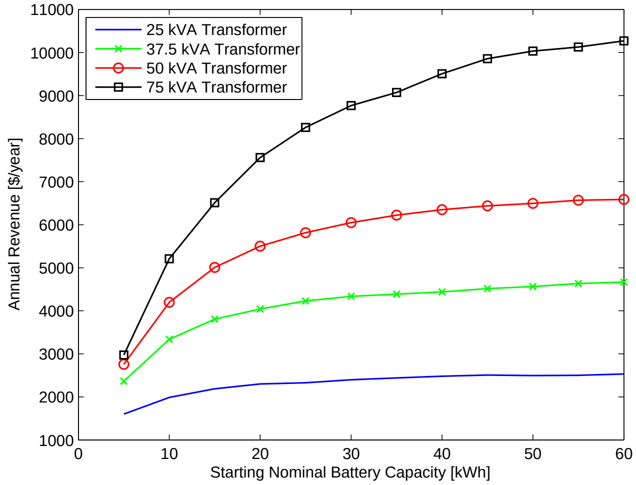


Fig Exp ct d annua batt ry op rating r v nu s

a arg r batt ry or transfor r giv s gr at r oss s sinc

Knitt CR Rob rts MR p An pirica xa ination of r structur d ctricity pric s En rgy Econo ics
Mohs ni P t vi RG E ctric v hic s Ho y grai or Foo s go d In Pow r En rgy oci ty G n ra M ting Institut of E ctrica and E ctronics Engin rs Ca gary AB pp p
Nasci nto JM Pow B An Opti a Approxi at Dyna ic Pro gra ing A gorith for th En rgy Dispatch Prob with Grid L v torag wor ing pap r
Nourai A Insta ation of th First Distribut d En rgy torag yst DE at A rican E ctric Pow r AEP ch R p AND p andia Nationa Laboratori s
Pow B Approxi at Dyna ic Progra ing o ving th Curs s of Di nsiona ity i y Int rsci nc Hobo n N w J rs y
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