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[illegible]

ited in that they do not fully account for the uncertain interactions between providing energy and AS.

The e□ects of price and system uncertainty are also often neglected in

3.3 State Variables

x : total energy in storage at the beginning of hour t [kWh]

3.4 Exogenous Variables

We assume that the variables p, p, p, D, I, δ, and δ evolve randomly and independently of any of the decision variables, but may be dependent on one another. We define $\hat{p}$, $\hat{p}$, $\hat{p}$, $\hat{D}$, $\hat{I}$, $\hat{\delta}$, and $\hat{\delta}$ as exogenous random more energy in net to the SO). Equation (2) defines the maximum amount of

Constraints (6), (7), and (8) together force the building load to be either served by the battery or left unserved in any time period with an outage. We let $\mathcal{A}$ denote the set of decision vectors, a, that are feasible in constraints (3) through (10) when the system is in the state s.

3.7 Objective Function

The net profit earned in hour t is given by:

C (S , a) = p (e

the solution. The pseudocode in Algorithm 1 summarizes the approximation algorithm that we use to find near-optimal solutions to our SDP. Steps 1 and 2 represent the first phase of the algorithm, in which the SDP is discretized and the resulting DSDP is solved exactly using backward induction. The second phase of the algorithm works by iterating through the hours of the optimization horizon. In each hour, the exogenous variables, W , are first observed (step 5). Then the amount of unserved regulation energy in hour $t-1$ is updated, based on the actual hour-$(t-1)$ dispatch-to-contract ratio (step 7) and the resulting energy level of the battery is determined (step 8). Finally, the resulting objective function value from using such a policy provides a statistical lower bound on the optimal value of the true SDP [29]. One of the lower bounds that we compute is found by randomly generating sample paths, ω, of the exogenous random variables, W, and using the approximation algorithm, outlined in Section 4.1, to derive a feasible policy. Because the approximation algorithm assumes that the hour $s > t$ exogenous random variables are unknown when hour-t

Algorithm 2 Backcasting Heuristic Pseudocode

1: Let $x_1 \leftarrow \underline{R}$ {assume battery starts empty}
2: for t = 1 to

We generate and solve 1,000 of these stochastic programs in order to compute a standard error for the upper bound.

Our other set of upper bounds is generated using a sample path averaging technique. This bound is computed by randomly generating sample paths of the exogenous random variables. For each sample path, ω , of exogenous state

also fit a log-normal distribution, which we assume. In order to capture diurnal energy and regulation price patterns, we allow for different location and scale parameters in the log-normal distributions for each of the 24 hours of the day. We fit these parameters using least-square estimation based on price data from the PJM market in the summer of 2009.

Seppala [28] examines the statistical properties of the electricity demand of residential homes. He compares several parametrized distribution functions and finds that a log-normal distribution provides the best fit. Thus, we assume that the building demand has a log-normal distribution and allow the location and scale parameters to vary in each of the 24 hours of the day. We fit these parameters using least-square estimation based on historical residential load data for the summer of 2009 provided by AEP for a set of its customers in Ohio. The loads correspond to a home that is approximately 200 m^2 (2200 ft^2) in size. Although there is a relationship between energy prices and loads, we are modeling a single building which has only a marginal effect on the system. Moreover, since we allow the distribution of the hourly prices and loads to vary, this captures any coincidence in energy prices and building demand.

The distributions of the dispatch-to-contract ratios for regulation up and down are estimated using least-square estimation based on historical PJM data from the summer of 2009. These data specify the amount of regulation capacity reserved in each hour and the amount of regulation energy deployed in real-time. The data do not show any diurnal patterns in the ratios. As such, we assume that the distributions are time-invariant. Hypothesis testing shows that a Gaussian distribution best fits the historical data, which we use in our case study.

A number of approaches are used to model power system reliability, with Markov-based models being the most common. The mechanics of component failure and repair suggest that power system failure follows a Markov process

Table 2 **Discretization of action variables in approximation algorithm.**

Variable	Number of Discretized Values
$\tilde{e}_t^d$	21
$\tilde{e}_t^c$	21
$\tilde{e}_t^l$	21
$\tilde{l}_t$	21
$\tilde{k}_t = \tilde{k}_t^u = \tilde{k}_t^d$	8

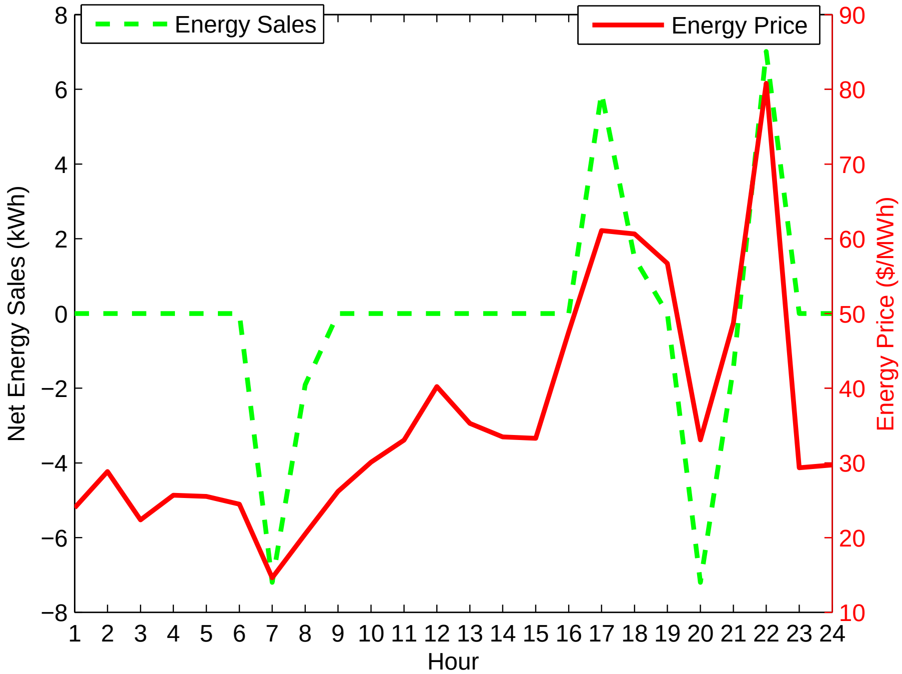
Energy Sales
Energy Price
Net Energy Sales (kWh)
Energy Price ($/MWh)
Hour
8
6
4
2
0
−2
−4
−6
−8
90
80
70
60
50
40
30
20
10
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24

Fig. 1

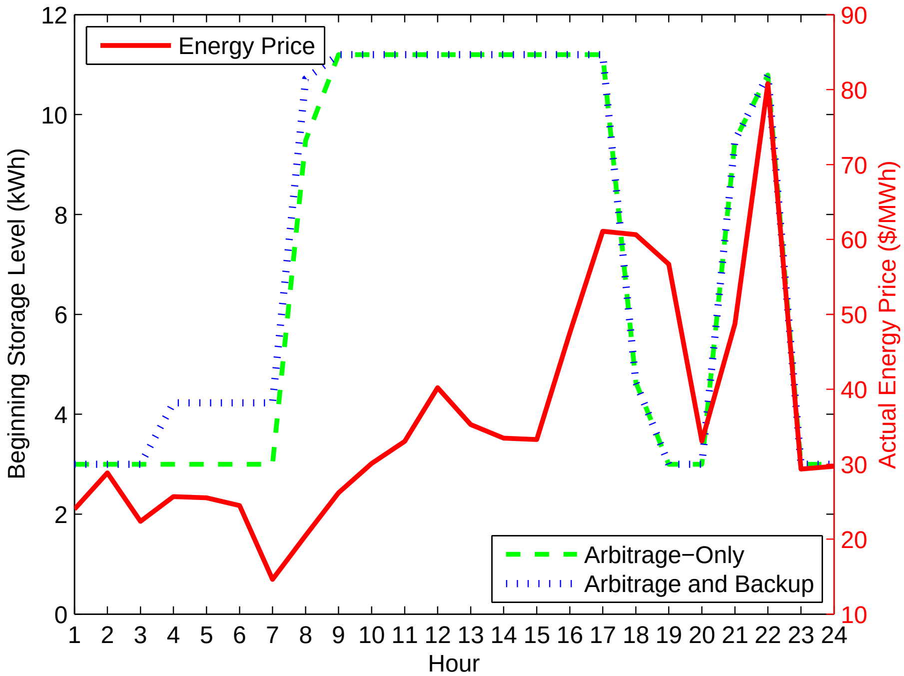

Fig. 2 Starting storage level of battery in arbitrage-only and arbitrage and backup-energy cases.

up to 0.35 in the historical PJM data—the regulation-up and -down signals tend to cancel out in the long run. Our simulation has high ratios of up to 0.30, but the average ratio over the week is much lower at -0.06, which is consistent with the historical PJM data. Thus, on average, providing regulation results in small net charging of the battery. This use of the battery reduces arbitrage profits even further compared to the other two cases—the battery earns $1.25 over the course of the week when regulation services are allowed—but the regulation profits of $23.90 more than compensate for this.

6.5 Solution Quality

Table

take. We can then define two sets of auxiliary variables, $z_{t+1}^{\zeta,j}$ and $v_{t+1}^{\zeta,j}$, where $\in$

References

1. Black, M., Strbac, G.: Value of Bulk Energy Storage for Managing Wind Power Fluctuations. IEEE Transactions on Energy Conversion 22, 197–205 (2007)
2. Brown, R.E., Gupta, S., Christie, R.D., Venkata, S.S., Fletcher, R.H.: Distribution system reliability assessment using hierarchical Markov modeling. IEEE Transactions on Power Delivery 11, 1929–1934 (1996)
3. Butler, P.C., Iannucci, J., Eyer, J.M.: Innovative Business Cases For Energy Storage In a Restructured Electricity Marketplace. Tech. Rep. SAND2003-0362, Sandia National Laboratories (2003)
4. van Casteren, J.F.L., Bollen, M.H.J., Schmieg, M.E.: Reliability assessment in electrical power systems: the Weibull-Markov stochastic model. IEEE Transactions on Industry Applications 36, 911–915 (2000)
5. Deane, J.P., Ó Gallachóir, B.P., McKeogh, E.J.: Techno-economic review of existing and

23. Mohseni, P., Stevie, R.G.: Electric vehicles: Holy grail or Fool's gold. In: Power & Energy Society General Meeting, 2009, pp. 1–5. Institute of Electrical and Electronics Engineers, Calgary, AB (2009)
24. Mokrian, P., Stephen, M.: A Stochastic Programming Framework for the Valuation of Electricity Storage. In: 26th USAEE/IAEE North American Conference. International Association for Energy Economics, Ann Arbor, MI (2006)
25. Nourai, A.: Installation of the First Distributed Energy Storage System (DESS) at