They also observe transmission congestion potentially increasing prices.

Although EPEC models are used to study market equilibria with renewables, interactions between renewables and energy storage are not well studied in the literature. This is particularly true of analyses of interactions between renewables and storage within a market equilibrium. This is because modeling energy storage requires a multi-period model to capture intertemporal constraints related to energy storage. Zou the appendices. More specifically, Appendix A shows how each firm's bilevel model is converted into an MPEC, by replacing the lower-level market operator's problem with necessary and sufficient primal/dual optimality conditions. Next, Appendix B shows how the firms' MPECs are combined to form an EPEC. Appendix C describes the steps that are taken to linearize the EPEC. Finally, Appendix D discusses how we verify that EPEC solutions are Nash equilibria.

3.1. Model Notation

We begin by first defining the following model notation. This includes sets and set-related parameters, model parameters, and lower- and upper-level variables.

3.1.1. Sets and Index Parameters

B — number of blocks for demand, generation, and storage bids and offers.
P — set of firms.
T — number of hours in model horizon.
Δ_p^G — set of conventional units owned by firm p.
Δ_p^S — set of storage units owned by firm p.
Δ_p^W — set of wind units owned by firm p.

3.1.2. Model Parameters

$C_{x,b}$ — marginal cost of generation block b of conventional unit x.
$\overline{D}_{t,b}$ — hour-t maximum demand in demand block b.
$\underline{E}_x$ — maximum storage capacity of storage unit x.
$G_{x,b}$ — capacity of generation block b of conventional unit x.
R_x^U — ramp-up limit of conventional unit x.
$\underline{R}_x^D$ — ramp-down limit of conventional unit x.
$\underline{S}_{x,b}^C$ — charging capacity of block b of storage unit x.
$S_{x,b}^H$ — discharging capacity of block b of storage unit x.
$\underline{U}_{t,b}$ — hour-t marginal utility of demand block b.
$W_{t,x,b}$ — hour-t available generation from block b of wind unit x.
η_x^C — charging efficiency of storage unit x.
η_x^H — discharging efficiency of storage unit x.

3.1.3. Lower-Level Variables

$D_{t,b}$ — hour-t demand of demand block b that is satisfied.
$E_{t,x}$ — ending hour-t storage level of storage unit x.
$G_{t,x,b}$ — hour-t dispatch of block b of conventional unit x.
$S_{t,x,b}^C$ — hour-t energy charged in block b of storage unit x.
$S_{t,x,b}^H$ — hour-t energy discharged from block b of storage unit x.
$W_{t,x,b}$ — hour-t dispatch of block b of wind unit x.

3.1.4. Upper-Level Variables

$O_{t,x,b}^C$ — hour-t bid price for charging block b of storage unit x.
$O_{t,x,b}^H$ — hour-t offer price for discharging block b of storage unit x.
$O_{t,x,b}^G$ — hour-t offer price for block b of conventional unit x.
$O_{t,x,b}^W$ — hour-t offer price for block b of wind unit x.

3.2. Market Operator's Problem

The market operator takes supply offers from the ge

exhibited in a 24-hour example. For instance, we have on- and off-peak load and wind periods. It is also worth noting that each of the time periods in the example could be used to represent a multi-hour block of time (*e.g.*, three-hour time periods). However, in our case study each period represents a single hour. Finally, it should be noted that our analysis is mostly focused on a qualitative assessment of market equilibria. That is to say, the exact values determined by the model are not as important as understanding how market equilibria compare to one another under different market and asset-ownership structures. Among the 24-hour case studies that we solve (for purposes of determining their solution times), we find that the market equiis o□ered as a price-maker. In Case 5 wind is o□ered at cost but storage can be o□ered at a price above its cost of zero.

5. Case-Study Results

Table 3 summarizes the results of the equilibria found for the di□erent cases using the two objective functions for the EPEC. Quasi-competitive equilibria see the highest possible demand levels being met. As such, the firms have very limited opportunity to exercise market power. Conversely, firms successfully restrict output and the amount of demand met to increase market prices in collusive equilibria. As such, Table 3 reports results for all nine collusive cases but only quasi-competitive Cases 1–5. This is because the wind generator is unable to exercise market power in the quasi-competitive equilibria and the results of Cases 6–9 are identical to those of Cases 2–5.

Adding wind to the market has the e□ect of suppressing prices. In Case 1, which has no wind, the average energy price is \$69.00/MWh and \$82.20/MWh in quasi-competitive and collusive equilibria, respectively. These prices are reduced to \$62.30/MWh and \$69.10/MWh, respectively, when wind is added in Case 2 . One way that wind generators can mitigate this price suppression is by

Table 3: Results of Market Equilibria

EPEC Objective Function	Case	Social Welfare [$]	Demand Met [%]	Firm Profits [$]		
				Conv.	Wind	Storage
Profit	1	89900	63.5	88700	n/a	n/a
Profit	2	128740	76.8	65090	45850	n/a
Profit	3	136195	74.6	65900	55310	3876
Profit	4	136031	75.2	68611	57490	629
Profit	5	136204	73.4	66514	57490	2900
Profit	6	122450	68.3	63680	53170	n/a
Profit	7	133125	71.4	65486	60706	1524
Profit	8	133820	71.4	66594	60926	1100
Profit	9	135870	73.0	66594	60064	2013
Welfare	1	94950	77.8	71250	n/a	n/a
Welfare	2	134510	89.8	59553	41937	n/a
Welfare	3	140547	88.3	40147	39170	4529
Welfare	4	140547	88.3	46920	39730	7697
Welfare	5	140547	88.3	57507	43010	7829

generators. Finally, we examine the increase in social welfare when storage is in the system, compared to a case without storage. This third case is examined because a policymaker may wish to incentivize investment in energy

Table 5: Justifiable Capital Cost of the Storage Unit Based on Increases in Profit or Social Welfare [$/kW]

		Basis of Calculation		
Equilibrium Type	Case	Storage Profit	Storage and Wind Profit	Social Welfare
Collusive	3	193	664	371
Collusive	4	31	611	363
Collusive	5	144	724	372
Collusive	7	76	451	531
Collusive	8	55	441	566
Collusive	9	100	443	668
Quasi-Competitive	3	225	88	300
Quasi-Competitive	4	383	273	300
Quasi-Competitive	5	390	443	300

base case, while the pairs of triangles indicate the ranges of these values obtained in the sensitivity cases. As expected, we find that social welfare decreases across the types of equilibria and market structures as generation cost increases and vice versa.

Changing generation cost has the expected effect on firm profits under collusive equilibria. Decreasing generation costs increases total firm profits while increasing costs has the opposite effect. Interestingly, this result does not necessarily hold under quasi-competitive equilibria. The reason for this is that quasi-competitive equilibria seek to maximize social welfare, which often entails higher levels of demand being served, as opposed to maximizing firm profits. As a result, some quasi-competitive equilibria with decreased generation costs result in more demand being served (to increase social welfare), which gives lower firm profits.

The amount of demand that is met is not adversely affected by increased generation costs under collusive equi-

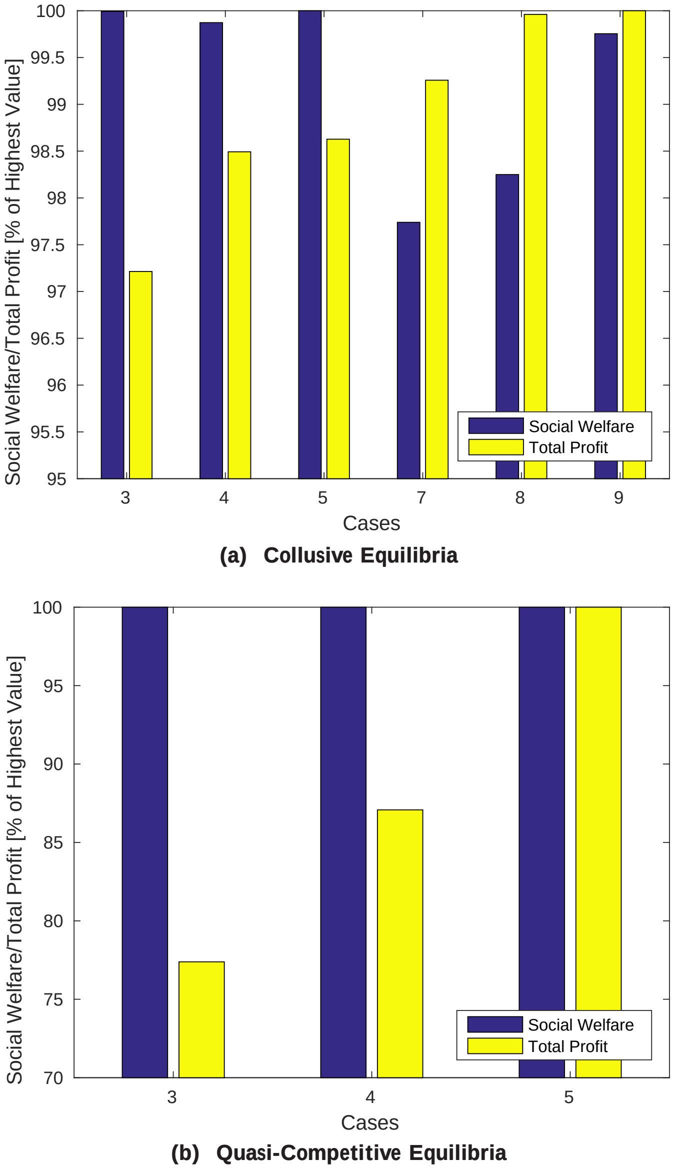


Figure 3: Social Welfare and Total Firm Profits Under Collusive and Quasi-Competitive Equilibria in Different Cases Examined [% of Highest-Value Case]

6.3. Wind Availability

Figures 10–12 summarize the range of changes in social welfare, total firm profits, and demand met (respectively) in a set of sensitivity cases in which wind availability is decreased or increased by 20% relative to its baseline value. As expected, social welfare, total firm profits, and demand met all remain the same or increase with greater wind

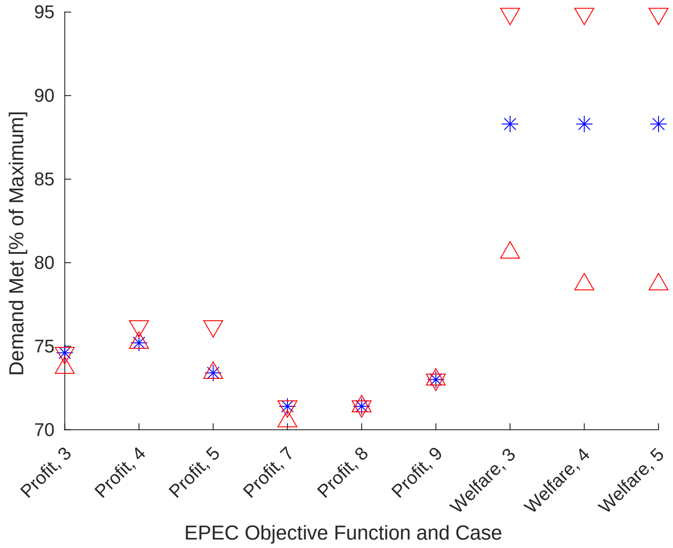
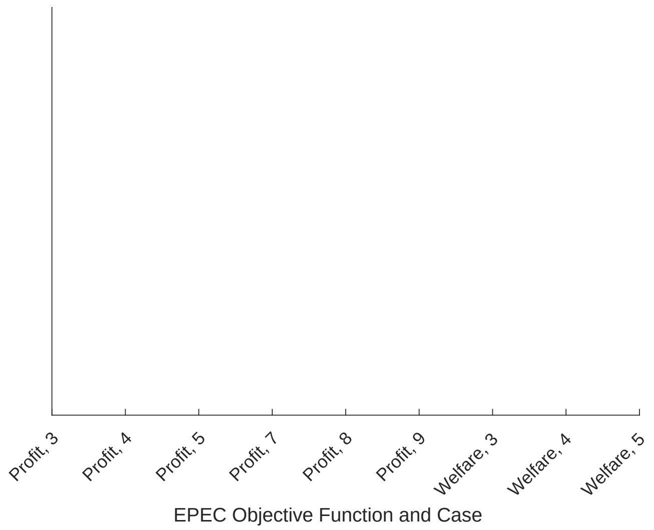
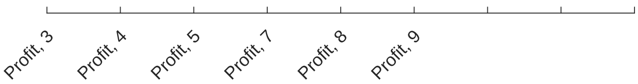


Figure 6: Range of Total Demand Met in Equilibria With Between 30% Decrease and 30% Increase in Conventional-Unit Costs Relative to Baseline (Star Indicates Baseline Value and Triangles Range of Values)




Profit, 3
Profit, 4
Profit, 5
Profit, 7
Profit, 8
Profit, 9

lem (*i.e.*, what we replace constraint (17) with) are:

$$\sum_{x,b} \left(\mathbf{G}_{t,x,b} + \mathbf{W}_{t,x,b} + \mathbf{S}^{H}_{t,x,b} - \mathbf{S}^{C}_{t,x,b}\right) = \sum_{b} \mathbf{D}_{t,b}, \quad \forall t \quad (\Psi_{p,t}) \tag{A.1}$$

$$0 \quad \mathbf{D}_{t,b} \quad \bar{\mathbf{D}}_{t,b}, \quad \forall t, b \quad (\Theta^{D,-}_{p,t,b}, \Theta^{D,+}_{p,t,b}) \tag{A.2}$$

$$0 \quad \mathbf{G}_{t,x,b} \quad \bar{\mathbf{G}}_{x,b}, \quad \forall t, x, b \quad (\Theta^{G,-}_{p,t,x,b}, \Theta^{G,+}_{p,t,x,b}) \tag{A.3}$$

$$-\mathbf{R}^{D}_{x} \quad \sum_{b} (\mathbf{G}_{t,x,b} - \mathbf{G}_{t-1,x,b}) \quad \mathbf{R}^{U}_{x}, \quad \forall t, x \quad (\Theta^{R,-}_{p,t,x}, \Theta^{R,+}_{p,t,x}) \tag{A.4}$$

$$0 \quad \mathbf{W}_{t,x,b} \quad \bar{\mathbf{W}}_{t,x,b}, \quad \forall t, x, b \quad (\Theta^{W,-}_{p,t,x,b}, \Theta^{W,+}_{p,t,x,b}) \tag{A.5}$$

$$0 \quad \mathbf{S}^{C}_{t,x,b} \quad \bar{\mathbf{S}}^{C}_{x,b}, \quad \forall t, x, b \quad (\Theta^{C,-}_{p,t,x,b}, \Theta^{C,+}_{p,t,x,b}) \tag{A.6}$$

$$0 \quad \mathbf{S}^{H}_{t,x,b} \quad \bar{\mathbf{S}}^{H}_{x,b}, \quad \forall t, x, b \quad (\Theta^{H,-}_{p,t,x,b}, \Theta^{H,+}_{p,t,x,b}) \tag{A.7}$$

$$\mathbf{E}_{t,x} = \mathbf{E}_{t-1,x} + \sum_{b} \left(\eta^{C}_{x} \mathbf{S}^{C}_{t,x,b} - \mathbf{S}^{H}_{t,x,b} / \eta^{H}_{x}\right), \quad \forall t, x \quad (\Theta^{E}_{p,t,x}) \tag{A.8}$$

$$0 \quad \mathbf{E}_{t,x} \quad \bar{\mathbf{E}}_{x}, \quad \forall t, x \quad (\Theta^{E,-}_{p,t,x}, \Theta^{E,+}_{p,t,x}) \tag{A.9}$$

$$\mathbf{E}_{T,x} = \mathbf{E}_{0,x}, \quad \forall x \quad (\Theta^{E,0}_{p,x}) \tag{A.10}$$

$$\mathbf{U}_{t,b} - \psi_{t} + \theta^{D,-}_{t,b} - \theta^{D,+}_{t,b} = 0, \quad \forall t, b \quad (\omega^{D}_{p,t,b}) \tag{A.11}$$

$$-\mathbf{O}^{G}_{t,x,b} + \psi_{t} + \theta^{G,-}_{t,x,b} - \theta^{G,+}_{t,x,b} + \theta^{R,-}_{t,x} - \theta^{R,-}_{t+1,x} - \theta^{R,+}_{t,x} + \theta^{R,+}_{t+1,x} = 0, \quad \forall t < T, x, b \quad (\omega^{G}_{p,t,x,b}) \tag{A.12}$$

$$-\mathbf{O}^{G}_{T,x,b} + \psi_{T} + \theta^{G,-}_{T,x,b} - \theta^{G,+}_{T,x,b} + \theta^{R,-}_{T,x} - \theta^{R,+}_{T,x} = 0, \quad \forall x, b \quad (\omega^{G}_{p,T,x,b}) \tag{A.13}$$

$$+ \psi_{t} + \theta^{G,}_{tb} - {}^{D,+}_{p,t,b}$$

$$0 \quad \bar{\mathbf{G}}_{x,b} - \mathbf{G}_{t,x,b} \quad \boldsymbol{\theta}^{G,+}_{t,x,b} \quad \mathbf{0}, \quad \forall \mathbf{t}, \mathbf{x}, \mathbf{b} \tag{C.4}$$

$$0 \quad {}^{-}_{b} \; (\mathbf{G}_{t,x,b} - \mathbf{G}_{t-1,x,b}) + \mathbf{R}^{D}_{x} \quad \boldsymbol{\theta}^{R,-}_{t,x} \quad \mathbf{0}, \tag{C.5}$$

$\forall \mathbf{t}, \mathbf{x}$

$$0 \quad \mathbf{R}^{U}_{x} - {}^{-}_{b} \; (\mathbf{G}_{t,x,b} - \mathbf{G}_{t-1,x,b}) \quad \boldsymbol{\theta}^{R,+}_{t,x} \quad \mathbf{0}, \tag{C.6}$$

$\forall \mathbf{t}, \mathbf{x}$

$$0 \quad \mathbf{W}_{t,x,b} \quad \boldsymbol{\theta}^{W,-}_{t,x,b} \quad \mathbf{0}, \quad \forall \mathbf{t}, \mathbf{x}, \mathbf{b} \tag{C.7}$$

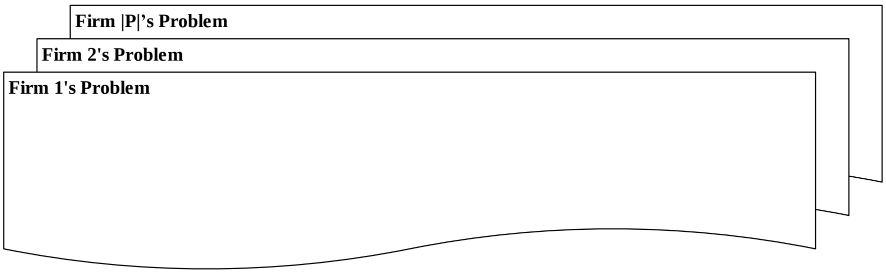
$$0 \quad \bar{\mathbf{W}}_{t,x,b} - \mathbf{W}_{t,x,b} \quad \boldsymbol{\theta}^{W,+}_{t,x,b} \quad \mathbf{0}, \quad \forall \mathbf{t}, \mathbf{x}, \mathbf{b} \tag{C.8}$$

$$0 \quad \mathbf{S}^{C}_{t,x,b} \quad \boldsymbol{\theta}^{C,-}_{t,x,b} \quad \mathbf{0}, \quad \forall \mathbf{t}, \mathbf{x}, \mathbf{b} \tag{C.9}$$

$$0 \quad \mathbf{0}$$

Appendix D. Verification of Equilibria

The linearizations that are outlined in Appendix C allow us to convert the EPECs into MILPs. However, a solution to an EPEC is not necessarily a Nash equilibrium. Rather, it is a point that satisfies the KKT condi-


Firm |P|'s Problem
Firm 2's Problem
Firm 1's Problem