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Towards Equilibrium Offers in Unit Commitment Auctions with Nonconvex Costs

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Abstract We compare two types of uniform-price auction formats commonly used in wholesale electricity markets—centrally committed and self-committed markets. Auctions in both markets are conducted by an independent system operator that col-

1 Introduction

Wholesale electricity markets facilitate the trade of electricity across a system of transmission lines. Such markets often use uniform-price auctions to determine the price of electricity, and the generators that submit the lowest bids, or equivalently offer to produce electricity at the lowest price, are selected to produce electricity. The two key outcomes of the auction process are generator commitment (which generators startup), and generator dispatch (the amount of electricity each generator produces). Independent system operators (SOs) conduct the uniform-price auctions repeatedly throughout the day.

A debate exists as to which entity, the SO or the generators themselves, should make these decisions. In centrally committed markets, generators submit two-part bids, subject to offer caps, and the SO makes the commitment and dispatch decisions and guarantees that each generator recovers the startup costs stated in its energy offer. This guarantee is made through a make-whole payment, which is a supplemental payment given to a generator for any deficit between its as-bid cost and energy payments. In a self-committed market each generator makes its own commitment decision and submits a single-part bid for energy, also subject to an offer cap, and must incorporate its startup costs into this bid.[1]

An unresolved issue in wholesale electricity market design and regulation is what equilibrium bidding behavior, the total cost of electricity service, and system efficiency would be under central and self commitment. This design question is important, given the considerable size of the markets.[2] The revenues in these markets also have significant implications for investment in new generation capacity, which determines the future electricity costs. The debate over the two market designs centers on the tradeoff between efficient dispatch and commitment, and generator incentives to truthfully reveal startup and energy costs. [Ruff (1994), Hogan (1994), Hogan (1995), Hunt (2002)] support centrally committed markets because they give the SO, which has the best information about the electric system as a whole, the authority to make both commitment and dispatch decisions. However, [Oren and Ross (2005)] show that generators can have incentives to misstate their costs to increase profit if the SO collects multi-part bids. Moreover, [Johnson et al (1997), Sioshansi et al (2008)] claim that incentive compatibility issues in a centrally committed market can be further exacerbated if the SO must rely on suboptimal solutions to its unit commitment model. As such, [Wilson (1997), Elmaghraby and Oren (1999)] suggest that commitment decisions are ultimately more efficient in self-committed markets.

Despite the various claims about the two market designs, their incentive properties have not been directly compared. To this end, we develop a single-period symmetric duopoly model of two markets: a centrally committed market with two-part offers (energy and startup); and a self-committed market with one-part offers (energy only). By analyzing the market as a uniform-price auction with system-wide caps on

[1] Some electricity markets operate as a hybrid between the two designs highlighted here. For instance, the New York ISO incorporates some non-convex costs, such as startup costs, into the energy price.

[2] According to their 2007 Annual Reports, the sum of wholesale transactions in 2007 were: $30.5 billion in PJM Interconnection, $9.5 billion in New York ISO, $10 billion in ISO New England, and $1.9 billion in ERCOT.

each bid element, we are able to characterize Nash equilibria in each market. We further derive conditions on the offer caps in the two markets that will yield expected cost equivalence between the two market designs. We also use a numerical example to demonstrate and compare the nature of the equilibria of the two markets. The re-

If $l \leq K$ only one generator needs to be committed and dispatched to serve load, which will be the one with the bid that produces l MWh at lowest total cost. The expected quantity sold by generator i is thus given by:

$$q_i^c(\omega_i, \omega_j, l \;\; = \begin{cases} \min\{l, K\}, & \text{if } \sigma_i - l\varepsilon_i < \sigma_j - l\varepsilon_j \text{ and } l \leq K; \\ \frac{1}{2}\min\{l, K\}, & \text{if } \sigma_i - l\varepsilon_i = \sigma_j - l\varepsilon_j \text{ and } l \leq K; \\ 0, & \text{if } \sigma_i - l\varepsilon_i > \sigma_j - l\varepsilon_j \text{ and } l \leq K; \end{cases}$$

and the uniform price of energy is set based on the ε of the generator that is committed and dispatched. We assume that ties are broken with equal probability. Conversely if $l > K$, both generators must be committed and dispatched and the quantity sold by the generators will be based on energy cost only. Thus generator i's expected production is:

$$q_i^c(\omega_i, \omega_j, l \;\; = \begin{cases} K, & \text{if } \varepsilon_i < \varepsilon_j \text{ and } l > K; \\ \frac{1}{2}l, & \text{if } \varepsilon_i = \varepsilon_j \text{ and } l > K; \\ l - K, & \text{if } \varepsilon_i > \varepsilon_j \text{ and } l > K; \end{cases}$$

and the uniform energy price is $p = \max\{\varepsilon_i, \varepsilon_j\}$.

In both cases, the generators receive energy payments, $p \cdot q_i^c(\omega_i, \omega_j, l$. However, the generators have non-convex costs due to their startup cost, so these energy payments alone may be confiscatory. The only information the SO has about the costs of the generators is their ‘as-bid’ costs in ω

non-negative and that there is a cap, δ^*, below which the bids must be. Given the

Proof When $l \le K$ the unique generator will be dispatched to serve the entire load, l, and the uniform price for energy is $p = \varepsilon_U$. Since the startup cost in its offer is non-negative, $\sigma_U \ge 0$, the unique generator's surplus from energy payments according to as-bid costs is $\varepsilon_U l - (\varepsilon_U l + \sigma_U) \le 0$. Thus the make-whole payment will be $W_U = \max\{0, \sigma_U + l(\varepsilon_U - \varepsilon_U)\} = \sigma_U$. Hence, the unique generator's total payment is $T_U = \varepsilon_U l + \sigma_U$.

When $l > K$ the marginal generator will be dispatched to serve $(l - K)$ units of the load and the uniform price is $p = \varepsilon_M$. Again, since $\sigma_M \ge 0$, the marginal generator's as-bid surplus from energy payments will be non-positive, thus the total payments will be the sum of energy and make-whole payment, hence $T_M = \varepsilon_M(l - K) + \sigma_M$, where the make-whole payment is $W_M = \sigma_M$.

Moreover, because of the make-whole provision, the SO will ensure the inframarginal generator's as-bid surplus is $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\}$. If $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\} = (\varepsilon_M - \varepsilon_I)K - \sigma_I$, then $\varepsilon_M K \ge \varepsilon_I K + \sigma_I$ and the total payment to the inframarginal generator is simply the energy payment, $\varepsilon_M K$, because the energy payment alone is sufficient to cover the inframarginal generator's (as-bid) startup and variable operating costs. Otherwise, if $\max\{(\varepsilon_M - \varepsilon_I)K - \sigma_I, 0\} = 0$ then $\varepsilon_M K < \varepsilon_I K + \sigma_I$, and the total payment to the inframarginal generator is:

$$
\begin{aligned}
T_I &= pK + W_I \\
&= \varepsilon_M K + \max\{0, \sigma_I + K(\varepsilon_I - \varepsilon_M)\} \\
&= \varepsilon_I K + \sigma_I,
\end{aligned}
$$

which is the desired expression.

Having characterized generator payments under the centrally committed market, we now prove the following result, which gives the set of Nash equilibria when only one of the generators is needed to serve the load.

Proposition 1 *If $l \le K$, the unique set of pure-strategy Nash equilibria of the centrally committed market consists of offers such that $\omega_i \in B$ for $i = 1, 2$, where B is the set:*

$$B = \left\{(\varepsilon, \sigma) \in \mathbb{R}^2 \mid \varepsilon l + \sigma = cl + S,\ \varepsilon \in [0, \varepsilon^*],\ \textit{and}\ \sigma \in [0, \sigma^*]\right\},$$

and each generator has an expected profit of zero.

Proof Given that $l \le K$, the SO only needs to commit and dispatch one generator and the SO does so in the least-costly way. Thus, the SO selects the generator with the lowest total cost. The dispatch is determined by the ranking of these costs, which for simplicity we refer to as $b_i = \varepsilon_i l + \sigma_i$ for $i = 1, 2$. This game is thus isomorphic to a simple Bertrand game, but in this case, each generator submits a total cost $b_i = \varepsilon_i l + \sigma_i$. The total cost of each generator, b_i is such that $b_i = cl + S$ for $i = 1, 2$ and generators earn zero profit in equilibrium. Clearly, there are many ω that belong to the set B but all vectors are payoff-equivalent because they result in the same expected commitment, dispatch, and profits. Moreover, since the total cost of the offers equal actual costs, expected profits are zero in equilibrium.

We now turn to the case in which $l > K$ and both generators must be committed and dispatched to serve the load. Since both generators must be committed, their startup costs must be borne, thus the optimal commitment and dispatch decisions will be made purely on the basis of each generator's energy offer, ε. As we show in the following lemmas and propositions, this characteristic of an optimum, coupled with the generators' binding capacity constraints, eliminates the possibility of a pure-strategy Nash equilibrium in the bidding game. As such, we assume that the generators follow mixed-strategy equilibria. This, in turn, implies that each generator has a strictly positive probability of receiving make-whole payments, and as such each generators' expected profit function is a non-decreasing function of its startup bid. Thus, each generator will submit an offer with a startup cost equal to the startup offer cap, σ^*.

Proposition 2 *If $l > K$, no pure-strategy Nash equilibria exist in the centrally committed market.*

Proof Suppose $(\tilde{\varepsilon}_i, \tilde{\sigma}_i$, for $i = 1, 2$, constitute a pure-strategy Nash equilibrium, and assume without loss of generality that the generators have been labeled such that $\tilde{\varepsilon}_1 \leq \tilde{\varepsilon}_2$.

Suppose first that $\tilde{\varepsilon}_1 < \tilde{\varepsilon}_2$. Then generator 1 is the inframarginal generator and its profit is:

$$\tilde{\Pi}_1 = \max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K - \tilde{\sigma}_1\} - cK - S.$$

If $\max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K - \tilde{\sigma}_1\} = \tilde{\varepsilon}_1 K - \tilde{\sigma}_1$ then generator 1 can profitably deviate by changing the energy portion of its offer to $\hat{\varepsilon}_1 = \tilde{\varepsilon}_2 - \eta$, with $\eta > 0$ and small, since its profits are increasing in ε_1. If, instead, $\max\{\tilde{\varepsilon}_2 K, \tilde{\varepsilon}_1 K - \tilde{\sigma}_1\} = \tilde{\varepsilon}_2 K$ then generator 1 can profitably deviate by changing its offer to $(\hat{\varepsilon}_1, \hat{\sigma}$

Lemma 2 *If* $l >$

or as:

$$f(\varepsilon) = \frac{F(\varepsilon)}{c-\varepsilon} - \frac{F(\varepsilon - \sigma^*/K)K}{(l-2K)(c-\varepsilon)}, \tag{3}$$

since the equilibrium is symmetric.

Equation (3) is a differential difference equation (DDE) characterizing a symmetric Nash equilibrium energy offer density function. We can find a particular solution of the DDE if we specify an interval of boundary conditions of width (σ^*/K). We do this by showing that the common supremum of the Nash equilibrium CDFs must be the offer cap, ε^*, which implies that $F(\varepsilon) = 1$ for all $\varepsilon \geq \varepsilon^*$.

Lemma 7 *If $l > K$, then a Nash equilibrium energy offer density function must have $\overline{\varepsilon} = \varepsilon^*$:*

Proof Suppose that $\overline{\varepsilon} < \varepsilon^*$ in an equilibrium. Then generator j has a profitable deviation whereby it moves the density assigned to the interval $(\overline{\varepsilon} - \eta, \overline{\varepsilon})$ to an energy offer of ε^*, with $\eta > 0$. We can bound the change in generator j's expected profits depending on whether it would be the marginal or inframarginal generator with the original strategy and deviation:

- If generator j is the inframarginal generator and would have been the inframarginal generator without deviating, its expected profits will either increase by at least $(\varepsilon^* - \overline{\varepsilon})(l - K)$ if it receives make-whole payments or will not change if it does not receive make-whole payments.
- If generator j is the marginal generator and would have been the marginal generator without deviating, the deviation will increase the pr0l will tdwe be.96264 Tf 171.12 0 Td [(j)0.963071]TJ /R16 9.96264 Tf 3 0 T

3.2 Self-Committed Market Equilibrium

or

$$g(\delta \;\; - \lambda \frac{G(\delta}{\delta - c} = 0, \tag{8}$$

where we have dropped the subscripts, due to the symmetry of the equilibrium, and defined $\lambda = (l - K \;\; /(2K - l \;\;$. The differential equation (8) can be solved by defining the integrating factor:

$$\begin{aligned} \mu(\delta \;\; &= \exp\left\{ -\int_a^\delta \frac{\lambda}{\tau - c} d\tau \right\} \\ &= \left(\frac{\delta - c}{a - c} \right)^{-\lambda}, \end{aligned}$$

where a is an arbitrary constant. Multiplying both sides of equation (8) by $\mu(\delta \;\;$ and integrating with respect to δ yields:

$$\begin{aligned} G(\delta \;\; &= b \exp\left\{ \int_a^\delta \frac{\lambda}{\tau - c} d\tau \right\} \\ &= b \left(\frac{\delta - c}{a - c} \right)^{\lambda}, \end{aligned}$$

where b is a constant of integration. In order to specify an exact solution to the differential equation we use the boundary condition that neither generator has a mass point at the supremum offer, δ^*, hence $G(\delta^* \;\; = 1$ which gives:

$$b \left(\frac{\delta^* - c}{a - c} \right)^{\lambda} = 1 \Longrightarrow b = \left(\frac{a - c}{\delta^* - c} \right)^{\lambda} \Longrightarrow G(\delta \;\; = \left(\frac{\delta - c}{\delta^* - c} \right)^{\lambda},$$

which is the CDF of the mixed-strategy Nash equilibrium.

We can also derive expressions for the expected energy offer and expected profit

expected settlement costs. Although the two markets are exp

5 Discussion and Conclusion

bids every six months. Furthermore, regulators often empower SOs to conduct market mitigation, whereby they can scrutinize bids that seem excessively high or uncompetitive. These types of factors are not included in our analysis either, which is reflected in the nature of the equilibria that we derive. For instance, the pure-strategy Nash equilibria that we find in the self-committed market would likely lead to scrutiny, and