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Working Memory: Activation Limitations on Retrieval*

JOHN R. ANDERSON, LYNNE

Carnegie Mellon University

Two experiments which require subjects to hold a digit span while solving an equation and then recall the digit span are performed. The size of the memory span and the complexity of the equation are manipulated as well as whether the subject is required to substitute items from the digit span for constants in the equation. As either task (digit span recall or equation solving) gets more complex there are performance decrements (accuracy or latency) not only in that task but also in the other task. It is also shown that the majority of the errors are misretrievals. These results are consistent with the proposal that working memory load has its impact on retrieval from memory. These results are fit by the ACT-R theory (Anderson, 1993) which assumes that there is a limit on source activation and that this activation has to be divided between the two tasks. As either task increases in complexity there is less activation for retrieval of information from declarative memory. Subjects' misretrievals of associatively related information could be predicted by assuming a partial matching process in ACT-R. q 1996 Academic Press, Inc.

As Baddeley (1992) notes there are several senses in which the term *working memory* has been used. The paper will be concerned with two of these senses. One is associated with the tradition that defines working memory in terms of paradigms which require the subject to maintain a memory load while performing a task (e.g., Baddeley & Hitch, 1974; Daneman & Carpenter, 1980). The second is associated with production system theories (e.g., Newell, 1991) where working memory is taken to be the currently available information against which production rules match. We are interested in relating these two senses because the ACT theory (Anderson, 1976, 1983, 1993) is associated with both. The ACT theory is associated with the first because of its strong roots in the human memory literature. It is associated with the second because it is a production system theory. ACT is a bit peculiar as a production system theory in that it does not have a working memory as that term is usually understood in production systems. Rather, the concept of capacity limitations is carried by the concept of activation. Elements in declarative memory have activation levels associated with them and access to these elements is a function of their level of activation. Roughly, working memory

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can be equated with the portion of declarative memory above a threshold of activation. Up until now we have never explored in detail how experimental manipulations of working-memory load would impact ACT's activation-based performance.

We have speculated about the role of working-memory load in studies of the effects of task complexity on skilled performance (Anderson & Jeffries, 1985; Anderson, Reder, & Ritter, in preparation). We have found that certain errors occur more frequently in the presence of greater complexity. For instance, Anderson et al. examined the frequency of errors while solving alge-

PROCESS-COLUMN

IF the goal is to write out an answer in column c1
and d1 and d2 are digits in that column
and d3 is the sum of d1 and d2
THEN set a subgoal to write out d3 in c1.

The first clause in this production matches the current goal to process the tens column; the second clause matches the digits in the tens column; and the third clause matches a fact or chunk from long-term memory. According to the ACT-R theory, an important component of the time for this production to apply will be the time to retrieve the long-term memories required to match the production rule. So, in this case where 3 and 4 are in the current column, the time to match the last clause will be determined by the level of activation of the chunk encoding 3 / 4 7 in Fig. 1. We explain how activation determines match time in the next subsection.

Activation

Activation of declarative structures has always been an important concept in the ACT theories. Basically activation determines how available information will be.[1] The activation of a chunk is the sum of source activation it receives from the elements currently in the focus of attention. Formally, the equation in ACT-R for the activation of element *i* is

$$A_i \quad \sum_j W_j S_{ji}, \tag{1}$$

where W_j is the salience or source activation of element *j* in the focus of attention, and S_{ji} is the strength of association from element *j* to *i*.[2] For instance, in the context of retrieving the chunk that 3 / 4

Gillund & Shiffrin, 1984) except that our activations are like logarithms of SAM familiarities since they add rather than multiply. It will prove important to keep conceptually separate the quantities A_i and W_j. The former are activations, which control retrieval from declarative memory, while the latter reflect the salience or attention given to the cues.[3] The W_j's are referred to as *source activations.*

The levels of activation determine the odds that a chunk will be retrieved and the time to perform that retrieval. These measures are described by equations of the form

$$\text{Odds}_i \quad Ce^{cA_i} \tag{2}$$

$$\text{Time}_i \quad Be^{0\,bA_i}, \tag{3}$$

where A_i is the level of activation of the chunk i, and C, c, B, and b are constants mapping A_i onto the two performance measures.[4] The underlying model is one in which chunks are retrieved as candidates to match a chunk pattern in a production until one is matched (producing a latency) or until a give-up time is reached (producing an error). The exponential functions in Eqs. (2) and (3) allow for the kind of nonlinear mapping of activation onto behavior required in many activation models (e.g., McClelland & Rumelhart, 1986; Rumelhart & McClelland, 1986). For a justification of the exponential assumption in ACT-R, see Anderson (1993).

Working-Memory Limitation

It remains to specify a theory of working-memory limitation that can be related to manipulations of task complexity. In the context of the current theory, the natural assumption is that there is some limitation on total source activation. Formally, this limitation is

$$\sum_j W_j \quad \text{Constant.} \tag{4}$$

This reflects a limitation on the amount of attention one can distribute over source objects. This is a new assumption, not specified in Anderson (1993). This paper will explore how well we can account for working-memory phenomena by making this assumption.

This resource limitation has some similarity to the ideas introduced by Kahneman (1973) and has quite a bit of similarity to the Just and Carpenter (1992) CAPS theory which interprets working-memory limitation as a limitation on the total amount of activation available in a production-system archi-

[3] The W_j

tecture. However, there are differences with the CAPS theory. Activation in the CAPS theory spreads by production firings rather than associations directly from sources to memory structures. Also the ACT-R limitation is not directly a limitation on activation but rather on the sources of activation. The total activation (A_i's in Eq. (1)) is a function of the strengths S_{ji} as well as the W_j. Finally and most important, our capacity limitation impacts retrieval from long-term memory.

Summary

It is worth reviewing the significant claims of this analysis of working-memory limitation:

1. The fundamental limitation is on amount of source activation (Eq. (4)).
2. This will impact on the activation of individual memory chunks (Eq. (1)).
3. This in turn will impact on probability and speed of successful retrieval (Eqs. (2) and (3)).

The unique aspect of this analysis of working-memory limitation is its localization of the limitation as impacting retrieval from declarative memory. We report research consistent with this localization. However, we do not mean to imply that there might not be other capacity limitations such as the rehearsal limitations in Baddeley's (1986) theory.

EFFECTS OF WORKING-MEMORY LOAD

One of the implications of the proposed extension (Eq. (4)) to the ACT-R theory is that there is a limited resource which is source activation. This would imply that two competing tasks, each of which required some source activation, would interfere with one another. This has been explored in experiments which require subjects to maintain a memory span concurrently while performing a primary task. Baddeley and Hitch (1974) found an interaction between memory span and complexity of the primary task (a reasoning task) such that there was a greater effect of primary task complexity at higher memory spans. Halford, Bain, and Maybery (1984) report such an interaction, both for performance of the primary task (an algebra-like task) and recall of the memory span. However, such interactions have not always been found (e.g., Evans & Brooks, 1981; Klapp, Marshburn, & Lester, 1983).

Carlson, Sullivan, and Schneider (1989) reported an experiment relevant to the issue of what determines whether there is a working memory interaction between a primary task and a concurrent memory load. We designed our experiments after their paradigm. During part of their experiment they presented their subjects with a memory span of three or six elements. The memory span involved the presentation of assignments of binary values to variables. In the three-span case subjects might be given A 1, B 0, C 1. While holding this memory span subjects were required to predict the output for a logic gate given a particular set of input values. Then they were probed for their memory of the span by being presented with a question of the form A 0 which they had to judge as correct or false. A critical manipulation in this experiment involved the relationship between the memory span and the judgment of the logic gate. In the **irrelevant** condition there was no relationship. In the other two conditions subjects knew they might need the information in the memory set to judge the gates. In the **access** condition, rather than seeing binary input to the gates subjects saw two variables and had to retrieve the values of these variables and predict what the gate would do for these values. In the **expect** condition, subjects thought they might see variables but in fact saw 0's and 1's as input.

Carlson et al. (1989) found little effect of size of memory span on irrelevant or expect trials but a large effect on access trials. The effect of three versus six memory load was 35 ms in the nonaccess conditions and 296 ms in the access condition. Also subjects were about 800 ms slower overall in the access condition. We were intrigued with this task for a number of reasons. First, Anderson (1989) argued that the large effect of access occurred because information in the memory span had to be used in the logic-gate task. Increased memory span would lower the activation of the individual elements in the memory span (as a fan effect) which would impact on the rate with which they could be used in the logic task. Thus, in effect, Anderson's argument was that there were separate working-memory limitations in the digit span and logic-gate tasks and the only way to get an effect of memory span was to integrate the memory span into the logic-gate task. This contrasts with Eq. (4) which proposes a single resource limitation.

We were also interested in the Carlson et al. (1989) manipulation because we thought this was a good way to explore the effects of working-memory

TABLE 1
Example Problems Used in Experiment 1

	No substitution		Substitution	
One transformation	3*x*	6	*ax*	*b*
	x/3	6	*x*/*a*	*b*
	3 / *x*	9	*a* / *x*	*b*
	3 0 *x*	9	*a* 0 *x*	*b*
Two transformations	3*x* / 2	8	*ax* / *b*	11
	3*x* 0 2	7	*ax* 0 2	*b*
	x/3 / 2	8	*x*/*a* / *b*	11
	x/3 0 2	7	*x*/3 0 *a*	*b*

the memory set in performing the algebra task. Table 1 illustrates the 16 types of material that were used. Half of the problems required one algebraic transformation to solve and half required two transformations. There were four basic types of one-transformation equations, involving multiplication, division, addition, or subtraction from both sides. The two-transformation equations consist of one multiplication or division and one addition or subtraction, giving four possible combinations. In the no-substitution condition, integers appeared in the equations whereas in the substitution condition the letters *a* and *b* replaced two of the integers. In the one-transformation condition both integers were replaced while in the two-transformation condition a random pair of the three integers were replaced. In the substitution condition, the

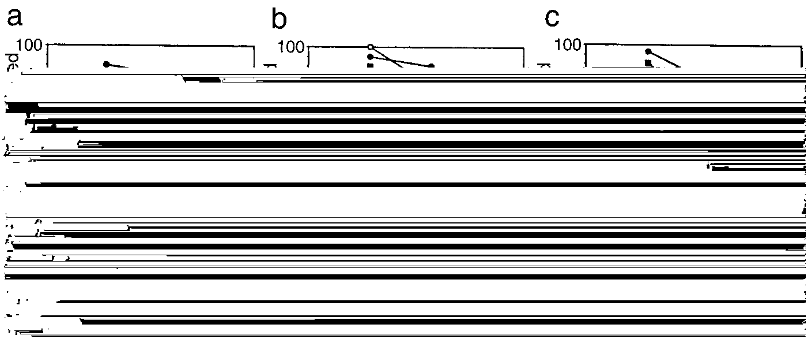

FIG. 2. Percentage of strings correctly recalled in Experiment 1: (a) Data; (b) Simulation; (c) Predictions of mathematical model.

it was replaced by an algebra problem. The subject solved the equation without benefit of paper

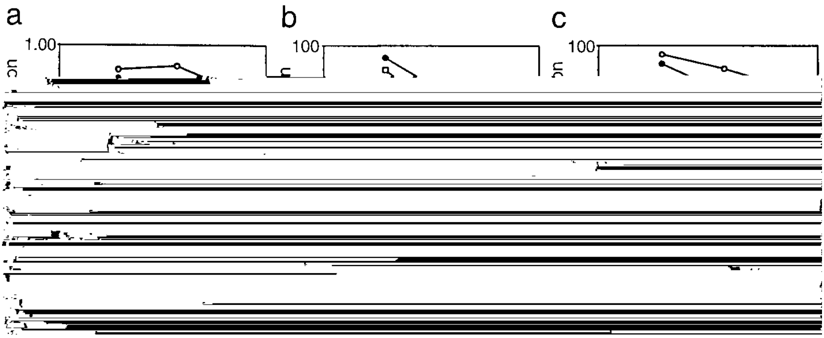

FIG. 3. Percentage of equations correctly solved in Experiment 1: (a) Data; (b) Simulation; (c) Predictions of mathematical model.

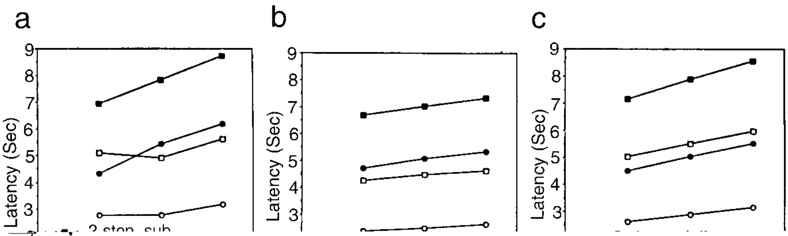
a
b
c
Latency (Sec)
Latency (Sec)
Latency (Sec)
9
8
7
6
5
4
3

TABLE 2
Productions Applying in the Solution of an Equation from Experiment 1

Initial state: *x*/*a* 0 *b* 4

Substitute for *a*:
IF the goal is to solve an equation
and ''*a*'' appears in the equation
and *f* is the first element of the memory set
THEN substitute ''*f*'' for ''*a*''

Resulting state: *x*/3 0 *b* 4

Substitute for *b*:

tic relationships, substitute the correct values, and type out the answer. Consider the most complex case which involved solving equations of the form

$$x/a \;\; 0 \;\; b \qquad 4$$

with a 3 and b 2. Table 2 gives the sequence of productions that applies in this case. Basically, there is one production for each operation in producing the answer. Our model for the recall of the memory span simply involved productions that encoded the incoming digits into successive serial positions and then retrieved from those positions.

Simulation

We ran a simulation of ACT-R which assumed fixed capacity for source activation which had to be divided between the terms in the equation and the terms in the digit span. This capacity was set at one unit.[5] We assumed that the capacity was equally divided among all of the symbols of the equation and the memory load. In the case above, there were seven symbols in the equation: x, /, a, -, b, , 4. Thus the total number of elements was 7 / s where s is the size of the digit span. Therefore, in the case of a two-transformation equation, the total activation of any element was 1/(7 / s). In the case of a one-transformation equation, this element activation was 1/(5 / s) because there were only five symbols in the equation.

Figures 2b, 3b, and 4b show the predictions from 125 Monte Carlo simulation runs per condition for the data in Figs. 2–4 using the parameters c 8, b 2, C 1, and B 1 for Eqs. (2) and (3). The two one-step curves in Fig. 3b are, by chance, identical.[6] The time scale in Fig. 4 is arbitrary and could be changed by changing the constant B. These simulations were run assuming the only time spent was in the retrieval involved in matching conditions. There is no cost for the action sides of productions. These simulations reproduce some of the qualitative appearance of the data; however, it is difficult to assess the goodness of this fit or to understand why the model fits in some places and misfits in other places.

One of the sources of complexity in understanding the simulation is that the activation levels vary with the strengths S_{ji} among elements (see Eq. (1)) which are a function of the exact connectivity among elements (see Anderson, 1993, for details). Also, there is a random component such that the results are only Monte Carlo approximations to the pure ACT-R predictions. In order to generate more precise predictions of the simulation, we produced a

[5] Since c and A

mathematical model of its application to this task and then optimized the fit of the mathematical model to the data. This is described below.

Mathematical Model

Retrievals from long-term memory are an important determiner of accuracy and latency in the ACT-R theory. Therefore, in developing a mathematical analysis of its predictions, it is important to identify which productions require retrieval from memory and how memory load impacts retrieval. Each production except *type-out* which is based on the earlier Eq. (2). The parameters *B*, *C*, *b*, and *c* were free to be estimated.

The time to complete a trial (solve the equation) is the sum of the amount of time associated with production firings, T_P, plus the component retrievals, T_R.

$$T(\text{Solution}) \quad mT_P \; / \; nT_R, \tag{7}$$

where *m* is the number of productions and *n* is the number of retrievals. In the example in Table 2, *m* 7 and *n* 6. Only the component T_R will be affected by memory span and activation. The component T_P reflects an ‘‘average’’ estimate of the time to do the other (non-long-term memory retrieval) matching of the production’s condition and to execute the production action.

As for accuracy, we assumed that all errors are caused by failure to correctly retrieve information. Thus, the probability of a correct answer is the product of the probabilities of correct retrievals:

$$P(\text{Solution}) \quad P_R^n, \tag{8}$$

where *n* is the number of retrievals. P_R will be impacted by activation which will in turn be a function of equation complexity and memory span. This assumes that no errors were due to systematic bugs in the subjects’ procedures. This seems a reasonable assumption given the high level of performance of all subjects on all equations.

Finally, we assumed memory span accuracy would simply reflect the accuracy of retrieval of the digits:

$$P(\text{String}) \quad P_R^S, \tag{9}$$

where *S* is the number of digits in the span. Note that it is the same P_R in lapsed over the substitution factor because the model predicts no effect of

TABLE 3
Sensitivity Analysis of the Model Fits for Experiments 1 and 2

	Original model	T_p 50% smaller	T_p 50% larger	B 50% smaller	B 50% larger	b 50% smaller	b 50% larger	C 50% smaller	C 50% larger	c 50% smaller	c 50% larger	b c
						Experiment 1						
T_p	0.74	0.37	1.11	0.76	0.83	0.52	0.84	0.74	0.74	0.74	0.74	0.82
B	1.88	1.65	0.00	0.94	2.82	1.48	2.80	1.88	1.88	1.88	1.88	2.51
b	3.16	1.31	1.37	1.81	4.68	1.58	4.73	3.15	3.15	3.15	3.15	4.34
C	7.38	7.38	7.38	7.38	7.38	7.38	7.38	3.69	11.07	19.82	2.88	7.63
c	4.42	4.42	4.42	4.42	4.42	4.42	4.42	6.01	3.51	2.21	6.62	4.34
x^2	25.05	26.12	40.56	27.28	25.21	25.57	25.22	27.19	25.91	30.73	28.9	25.16
						Experiment 2						
T_p	0.48	0.72	0.24	1.06	1.02	0.00	0.96	0.48	0.48	0.48	0.48	1.13
B	5.82	6.07	5.65	2.91	8.73	4.47	7.27	5.82	5.82	5.82	5.82	8.19
b	2.71	3.14	2.34	1.42	4.72	1.36	4.06	2.71	2.71	2.71	2.71	4.70
C	3.70	3.70	3.70	3.70	3.70	3.70	3.70	1.85	5.55	8.93	1.63	4.56
c	5.36	5.36	5.36	5.36	5.36	5.36	5.36	7.54	4.15	2.68	8.04	4.70
x^2	79.28	79.55	79.51	101.34	82.98	83.67	80.92	87.12	82.55	96.6	90.47	83.38

reports the results of these explorations. As can be seen, the quality of fit did not suffer much under these settings as compensating values could be estimated for the other parameters. The one exception was that the quality of fit decreased perceptibly when T_P was set to be 50% higher. The reason why these fits were so good generally is because B and b can trade off for latency and C and c can trade off for accuracy.[8] Larger values of the time scale parameter B can compensate for larger values of the exponent b and larger values of the odds scale parameter C can compensate for smaller values of the exponent c. It was apparent from this exploration that we could have a four-parameter version of this model in which the exponents are constrained to be the same ($b \quad c$) which is also reported in Table 3.

In summary, we think the model fits are sensitive to those aspects that were expected—the digit span (d), the symbolic complexity of the equation (s), and the relative number of memory retrievals (n) required for simple versus complex equations. With respect to the estimated parameters, the scale parameters B and C are being estimated to produce the average values observed of the latency and accuracy dependent measures. The exponents b and c are being estimated to produce the mapping of changes of activation onto changes in performance. Basically, the data are a function of how much the load produced by the combined tasks impacts the memory retrieval required in each task.

A Separate Capacity Model

We thought it would be informative to see how a model which assumed one capacity for digit span and a different capacity for equation solving would do at fitting these data. Thus, the activation available for doing the equation was $5/s$ and for the memory span $5/d$. Such a model without elaboration does poorly at fitting the data ($x^2 \quad 62.80$) since it fails to capture the task interactions. However, a reviewer pointed out to us that there was a fairly simple way to elaborate the model to produce some of the task interactions. There are two ways that the digit span might impact upon the equation solving despite the lack of shared capacity. First, in the case of substitutions, two retrievals from the span are required which will be impacted by the span activation. Second, it is possible that subjects were covertly rehearsing the span while solving the equation. (Although our subjects did not report doing this, research has shown it is rather difficult to assess implicit rehearsal; Reitman, 1974). More time would be taken away from equation solving for each digit that had to be rehearsed. Therefore, we estimated a mean time, r, for each second of equation solving that a subject would give to rehearsing a digit. Thus, if it took T s to solve the equation without a span and the span had d digits it would take $T*(1 \ / \ dr)$ s to solve the equation.

[8] Essentially what the exponent determines is how quickly changes in activation result in changes in time and accuracy, with larger values producing steeper functions while the scale parameters determine the average values of these functions.

Equation solving slowed the time before recall of the digits began and one might imagine that the activation of the digits decayed over this time. Assuming exponential decay, if it took T units to solve the equation, the activation would have decayed by an amount a^T where a is the fraction decayed each second. Thus, the parameter a becomes another parameter of the model.

We fit this seven parameter model to the data and achieved best fitting estimates of r 0.06 s, a 0.95, T_P 0.26, B 0.84 s, b 0.11, C 14.16, and c 1.83. The x^2 statistic was 36.14 which is about 10 larger than

TABLE 4
Example Problem Used in Experiment 2

	No substitution	Substitution
Simple	3*x* 7	*ax* *b*
	3 / *x* 7	*a* / *x* *b*
Complex	0 3/4*x* 7/2	0 (*a*/4)*x* *b*/2
	3/4 / *x* 0 7/2	*a*/4 / *x* *b*/2

complexity in number of symbols is correct, we should see larger interactions with equation complexity.

Table 4 illustrates the eight types of material that we used. All the problems were just one algebraic transformation removed from a solution. That transformation could involve either subtraction from both sides or division of both sides to isolate the variable. The arguments in the equation could be either simple positive integers or complex signed fractions. The fractional equations involve additional symbols for numerator, denominator, fraction bar, and number signs. Finally, the arguments could be numbers or the letters *a* and *b*

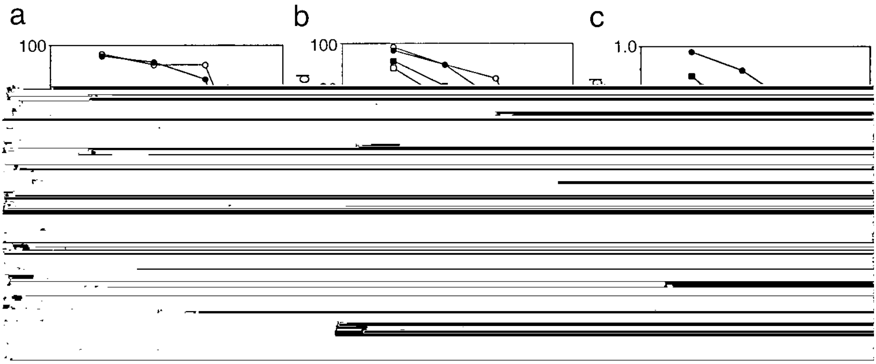

FIG. 5. Percentage of strings correctly recalled in Experiment 2: (a) Data; (b) Simulation; (c) Predictions of mathematical model.

a 2 1 2 1 4 analysis of variance where the factors were equation complexity, whether substitution was required, and memory span.

Figure 5a displays the results for percentage memory spans recalled. There were significant effects of size of memory span (F(3,57) 35.87, p .001) and of equation complexity (F(1,19) 46.75, p .001). The effect of substitution was not significant in this experiment as in Experiment 1 (F(1,19) 0.46). However, there was a substitution by complexity interaction (F(1,19) 9.03, p .01) such that subjects were 2% less accurate when they performed substitution for simple equations and 4% more accurate when they performed substitution for complex equations. The substitution effect for complex equations is significant (t_{19} 2.4, p .01). There is also a significant span-by-substitution interaction (F(3,57) 2.87, p .05) such that the substitution advantage is mainly for small spans. There were no other significant interactions. There is again a curvilinear trend in the effect of span: The decrease from two to six digits is significant (t_{57} 2.21, p .01) but significantly less than the decrease from six to eight (t_{57} 3.53, p .001). Thus, with respect to digit recall we need to account for the following facts:

1. There is an advantage of the substitution condition for complex equations with short spans.
2. There is an effect of equation complexity.
3. There is a larger effect of six versus eight digits than two versus six.

Figure 6a displays the results for percentage of equations correctly solved. There were significant effects of equation complexity (F(1,19) 47.85, p .001) and substitution (F(1,19) 20.00, p .001). The effect of memory span was marginally significant (F(3,57) 2.56, p .10). A specific contrast for a linear trend was significant (t_{57} 2.53, p .01). Since Experiment 1 found an effect of memory span, it seemed likely that there would be one

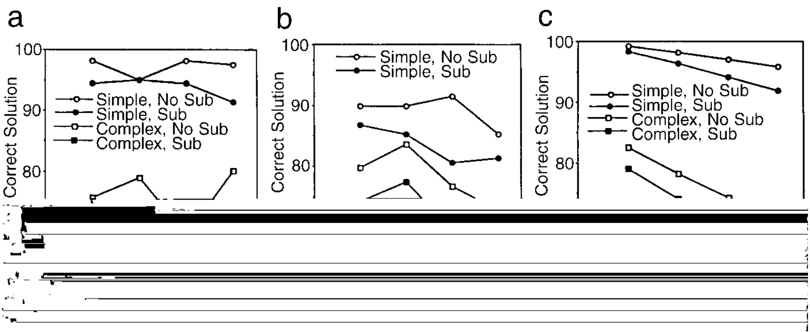

FIG. 6. Percentage of equations correctly solved in Experiment 2: (a) Data; (b) Simulation; (c) Predictions of mathematical model.

in this experiment too. This experiment also found a significant three-way interaction of complexity and substitution with memory span ($F(3,57)$ 5.91, p .005). The data are definitely noisy but fitting linear functions reveals no effect of span in the case of simple, no substitution and a 1.24% increase in error rate per item in the digit span in the case of the complex, substitution. This is the predicted three-way interaction that failed to be significant in the first experiment. Thus, with respect to accuracy of equation solving we need to account for the following facts:

4. There is a weak effect of memory span which is largest in the case of complex equations with substitution.
5. There is an effect of equation complexity.
6. There is an effect of substitution but smaller than the effect of equation complexity.

significant effects of equation complexity ($F(1,19)$ 4169.33, p .001), of memory span ($F(3,57)$ 10.65, p .001), and of substitution ($F(1,14)$ 110.27, p .001). There was also a significant interaction of memory span and substitution ($F(3,57)$ 5.74, p .01) such that the effect of memory span is greater in the case of substitution. This replicates the interaction found by Carlson et al. (1989). The increase in latency with increased memory span was only marginally significant in the case of no substitution (t_{57} 1.61, p .10) but quite significant with substitution (t_{57} 3.16, p .001). The two experiments both found marginal effects of span in the case of no substitution. Combining the two experiments, the effect is significant (z 2.19, p .01). Thus, unlike Carlson et al. we conclude that there is an effect of span on latency in the absence of the substitution requirement. In this experiment all the other interactions were significant as well—complexity by substitution ($F(1,19)$ 38.85, p .001); complexity by memory span ($F(3,57)$ 6.55, p .001); and complexity by substitution by memory span ($F(3,57)$ 4.89, p .001). This again is the predicted three-way interaction that failed to be significant in the previous experiment. In particular, the effect of memory span increases from simple, no substitution (slope 0.33 s per item) to simple, substitution (0.70 s per item), or complex, no substitution (0.69 sec per item) to complex, substitution (1.88 s per item). It is true that the effect of span tends to be larger in conditions with longer latency; however, this effect cannot be simply an artifact of larger effects for conditions with larger base RTs: the complex, no-substitution condition has the same slope as the simple substitution condition, yet the former has a much higher base RT. Thus, with respect to solution time, we need to account for the following effects:

7. The effect of memory span is larger in the case of substitution or in the case of complex equations.
8. There is an effect of complexity.
9. There is an effect of substitution but it is smaller than the effect of complexity.

The nine effects reported above substantially correspond to the results from the first experiment. Equation complexity, manipulated by use of fractions, produced effects similar to the effects of equation complexity, manipulated by number of transformations. Complexity in this experiment, though, produced larger effects particularly on the time to solve equations where subjects took almost five times longer to solve the complex equations.

Finally, almost as an aside, we note that there may be something of a speed-accuracy trade-off in the condition of solving complex equations with no substitution. This is the condition that produced the greatest deviations from monotonicity in Figs. 6 and 7. What is striking about these data is that they mirror each other—every time there is a dip or rise in accuracy (Fig. 6a), there is a compensating dip or rise in latency (Fig. 7a).

TABLE 5

Productions Applying in the Solution of an Equation from Experiment 2

Initial state: x / 3/4 0 7/6

Invert-transformation:

IF the goal is to solve an equation of the form ‘‘term op1 c d’’
where c and d are constants
and op2 inverts op1
THEN transform the equation to the form ‘‘term

ACT-R Model

We extended the ACT-R model from Experiment 1 to account for these data. The same sequence of production rules was used for the simple equations but a different and more complex set of productions was used for the complex equations to do the fractional arithmetic. Table 5 illustrates the sequence of productions that would be required to solve a problem involving addition of fractions.

This sequence for addition of fractions in Table 5 involves nine productions and six memory retrievals of arithmetic facts and one retrieval of an algebraic fact (*invert-transformation*) yielding seven memory retrievals. The corresponding sequence for multiplication involves seven productions and five memory retrievals (ignoring substitutions). In contrast, for simple equations there are always three productions and two retrievals for both addition and multiplication as in the past experiment. This contrast implies subjects should take longer to solve addition problems than multiplication problems for the complex equations but not for the simple equations. Indeed, there was an interaction between complexity and type of operation ($F(1,19)$ 38.85, p .001). In the case of simple equations there was no difference between addition and multiplication (6.25 versus 6.35 s) while addition took much longer in the case of complex equations (35.39 versus 23.29 s).

We ran the ACT-R simulation of this task 160 times per condition and the average data are illustrated in Figs. 5b, 6b, and 7b.[9] For the sake of brevity, we will proceed directly to describing the fit of the mathematical model of the application of the ACT-R theory to this task. In calculating activation sources, we assumed that each term in the equation plus each digit in the memory span was an element. Setting a bound on source activation of 1, as in Experiment 1, meant that each to-be-retrieved chunk had an activation of $5/(5 / s)$ in the case of simple equations and $5/(11 / s)$ in the case of complex equations where s was the number of digits in the memory span. Each fraction the value 79.3. This indicates a good fit but there is significant residual variance not predicted.

The fit of the mathematical model is shown in Figs. 5c, 6c, and 7c. Figure

Also as we did in Experiment 1, we performed a sensitivity analysis of our model. This is reported in the lower half of Table 3. As can be seen, the model is again relatively insensitive to the actual parameters estimated, trading off T_P, b, and B for predicting time and trading off c and C for predicting accuracy. Again, the model fits reasonably well with the constraint that the exponents b and c be equal.

The actual values estimated for these parameters are quite different in the two experiments. However, the trade-offs that exist suggest that we could constrain these estimates to be the same between the two experiments with relatively little effect on overall goodness of fit. We were able to fit the data of both experiments with the same set of parameters except that we needed to estimate two values of B, the time scale parameter. To see why this is necessary, consider performance on the simple equations with spans of 2 through 6 which were the common conditions in both experiments. For these comparable conditions there is little difference in accuracy between the two experiments: subjects solved 97% of the equations in Experiment 1 and 96% in Experiment 2; they recalled 95% of the spans in Experiment 1 and 96% in Experiment 2. On the other hand, there was a large difference in latency with subjects taking 4.1 s to solve these equations in Experiment 1 but 5.7 s in Experiment 2. Perhaps because Experiment 2 required a greater variety of facts to be retrieved, the time to retrieve any one fact was lower due to the less frequent repetition. We fit the experiments allowing two time scale parameters B_1 and B_2. The best fitting parameters were T_P 0.73 s, B_1 1.92 s, B_2 6.01 s, b

span with span size. For instance, Crannell and Parrish (1957) found a gradual drop off in accuracy as the digit span increases from 4 (nearly 100%) to 10 (nearly 0%). Unlike frequent popular characterizations, there does not appear to be a discontinuous ''drop-dead'' size. Indeed, typical span recall looks very much like that predicted by our model for the simple equations.

Because of their dual-task structure our experiments are not ideal if one's real goal was to study the nature of the memory span. As we noted in the previous experiment there is the possibility that subjects systematically allocate more of their capacity to the span and away from equation solving as the span gets larger. This would produce a flattening of the curve until high spans. There are other complications in the span task not accounted for by our model. These include effects of acoustic confusion (minimal for digits), time-based forgetting, and confusion among serial positions. The next section will show that serial position confusion was a significant factor in our experiment. Thus, our model for the span task in no way captures all of the complexities of what is occurring. The model is just complicated enough to accommodate the basic interactions between the processing demands of the two tasks.

We suspect that time-based forgetting was behind the interactions with substitution that we found in this experiment. Complex equations were taking on the order of 30 s to solve. Substitution offers an opportunity to rehearse the part of the span and this may have significant benefit in bridging this interval. This benefit would be greatest for short spans where substitution served to rehearse a significant fraction of the span. This may be why subjects showed a substantial advantage in Fig. 5 when they had to substitute from a short span for a complex equation.

NATURE OF ERRORS

The model as described so far has treated errors as resulting only from a failure of retrieval. One might assume that such failures would be just omissions. However, this assumption does not fit well with the observed errors. Subjects almost never failed to enter an answer to an equation

$$\text{term} \ / \ a \quad b \rightarrow \text{term} \quad b \ / \ a$$

$$\text{term} \ 0 \ a \quad b \rightarrow \text{term} \quad b \ 0 \ a$$

$$a{*}\text{term} \quad b \rightarrow \text{term} \quad a{*}b$$

$$\text{term}/a \quad b \rightarrow \text{term} \quad b/a$$

$$\text{term} \ / \ a \quad b \rightarrow \text{term} \quad b/a$$

$$\text{term} \ 0 \ a \quad b \rightarrow \text{term} \quad b{*}a$$

$$a{*}\text{term} \quad b \rightarrow \text{term} \quad b \ 0 \ a$$

$$\text{term}/a \quad b \rightarrow \text{term} \quad b{*}a.$$

If an error could not be classified as one of the above, it was classified as an arithmetic error if it could be produced by transforming a true addition fact $a \ / \ b \quad c$ or a true multiplication $a{*}b \quad c$ into a false fact by incrementing or decrementing one of the a, b, or c by 1 (e.g., $3 \ / \ 4 \quad 8$; $3 \ 1 \ 5 \quad 12$). In our opinion this classification underestimates the frequency of systematic errors since some appeared to result from multiple slips of this sort. Errors classified by our scheme occurred much more frequently than chance. We calculated chance by the following procedure: For each experiment, we took

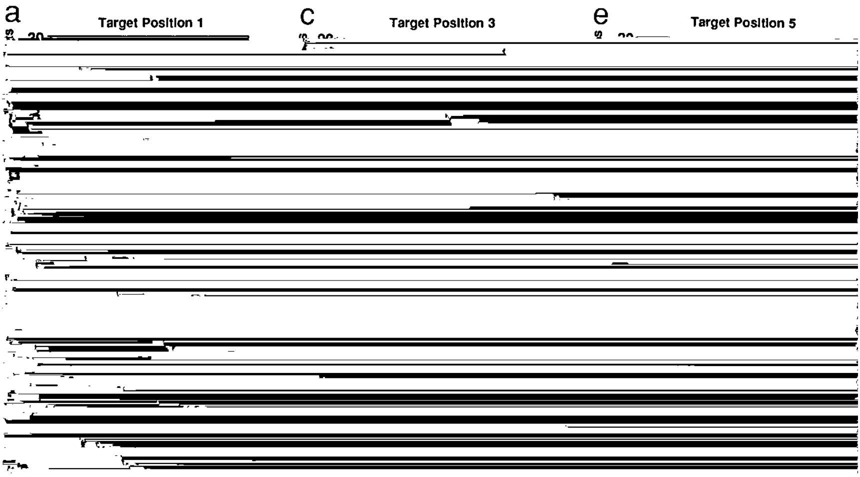

FIG. 8. Number of digits misplaced at each position for each target position.

and 2 to provide a representative illustration of these effects. Forty-six percent of the errors involved omission of the target digits. Such an omission could occur in a number of ways: Sometimes subjects did not recall the string at all, sometimes they recalled a fragment of the string, and sometimes they recalled a full six digit string but did not include the target digit. However, the other 54% of the time, the digit was recalled in the string but not in the right position. Figure 8 provides the data to show that these were not wild guesses and graphs the frequency of misplacements for each target position. As can be seen, there are positional uncertainty gradients such that subjects are most likely to misplace the digit in an adjacent position.

Partial Matching and Errors of Commission

The facts that error frequency was impacted by memory load and errors were misretrievals reinforces the localization of working-memory limitations in memory retrieval. The question remains of whether we can account for the exact nature of these errors in ACT-R. In ACT-R the most elegant way to account for these errors of commission is to allow chunks which are quite active but only partially match to be retrieved instead of the correct ones. Partial matching has received support, both empirically and computationally, in recent work of Reder (Kamas, Reder, & Ayers, in press, 1994; Reder & Cleeremans, 1990; Reder & Kusbit, 1991; Reder & Ritter, 1992; Reder, Schunn, Nhouyvanisvong, Richards, & Stroffolino, in preparation). There is this is decremented for each mismatch. Thus, if a chunk 3 / 5 8 is retrieved as an answer to a pattern for 3 / 4 ?, its match score is its activation minus a measure of the mismatch between 5 and 4. The degree of mismatch will be a function of the similarity between 4 and 5.

This scheme can be justified from a rational perspective if we allow that things do not have to match perfectly to be useful. Certainly, in matching real world things like faces which can vary in their dimensions and grow objects like mustaches, this makes sense. Only in highly formal domains like mathematics do things have to match perfectly to be used.

To get some variability in the responding we then added Gaussian noise to the activation levels. The odds formula in Eq. (2) was also based on the assumption of a Gaussian noise added to activation values which would cause them to sometimes fall below a threshold activation. In the current version, the Gaussian noise will occasionally cause the activation of the correct chunk to fall below the activation value of a distractor.

Let us first see how this mechanism can account for the pattern of arithmetic errors. When retrieving the sum of 2 and 5, both numbers are made sources and contribute activation to the correct fact: 2 / 5 7. Many other facts also receive activation from these and other sources and might gather more activation because of a Gaussian noise in the activation values. To favor close matches, similarity values between numbers are set to reflect their absolute difference.[12] Therefore, the penalty for 2 / 6 8 matching 2 / 5 will be less than the penalty for 2 / 1

Finally, we achieved good fits to the data within the ACT-R model which localizes the effect in memory retrieval.

In ACT-R the limitation is in source activation (Eq. (4)). This in turn limits the ability to get declarative chunks sufficiently active so that they can be retrieved or reliably discriminated from partially matching chunks. If we take working memory to mean the amount of declarative memory that ACT-R can reliably and quickly access, then limitations on source activation imply limitations on working memory. The ACT-R concept of working memory is rather nontraditional for production systems where working memory is normally thought of as some fixed set of information. The graded character of the effects in these experiments is clearly consistent with the ACT-R conception.

In addition to fitting the performance measures of error rate and latency, an extension of the ACT-R model was shown capable of accounting for the qualitative pattern of errors. Most of these errors were errors of commission and could be explained by incorrect retrieval of memories which were similar to the target memory and which were active in the experimental context.

It is interesting to try to characterize the implications of this research for Baddeley's (1986) model of working memory. One might try to map the separate-capacity model described earlier onto this theory, identifyt(on(E 0 0a

Finally, we want to comment on the research strategy reflected in this paper, which is to develop a simulation of a phenomenon, determine that it reproduces the basic qualitative character of the data, find a mathematical model of that simulation, and then optimize the fit of that model to the data to determine just how well the theory accounts for the results. We feel this reflects a powerful research strategy which is emerging in a number of efforts to test and develop large scale theories (e.g., McClelland, 1991). There is a real need to develop an integrated theory which is capable of accounting for a broad range of phenomena (Newell, 1991). Such theories offer the only real hope of transferring results from the laboratory to the real world where phenomena are not packaged into neat laboratory categories. On the other hand, there is a need to have such theories address the details of empirical phenomena that are the traditional tests of theoretical accuracy. By starting with a general-purpose simulation such as ACT-R, we achieve the desired broad generality. By producing a simulation for a specific task we achieve a detailed mapping of the theory to the experimental situation. By developing a mathematical model, we facilitate calculation of goodness of fit and identify

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