

ALGEBRA BASIC EXAM: JANUARY 2015

Attempt four of the following six questions. All questions carry equal weight. All rings are assumed to be commutative rings with 1.

- (1) Let the group  $G$  act on the set  $X$ . Denote the stabiliser of  $x \in X$  by  $G_x$ , the orbit of  $x$  by  $O_x$ , and the set  $\{x \in X : gx = xg\}$  by  $Fix(g)$ .
  - (a) State and prove the orbit-stabiliser relation.
  - (b) Now assume that  $G$  and  $X$  are finite.
    - (i) Prove that