

ALGEBRA BASIC EXAM: SEPTEMBER 2015

Attempt four of the following six questions. All questions carry equal weight. All rings are assumed to be commutative rings with 1, and all ring HMs are assumed to preserve 1.

- (1) State the Sylow theorems. Let G be a finite group, let $H \subset G$ and let P be a Sylow p -subgroup of H . Prove that $G = HN_G(P)$.
- (2) Define the terms *algebraic extension*, *separable extension*, *normal extension*, *splitting field*, *Galois extension*. Prove or disprove (by a counterexample) the following statement: if E_1 is a separable extension of E_0 and E_2 is a separable extension of E_1