

ALGEBRA BASIC EXAM: JAN 2016

Attempt four of the following six questions. All questions carry equal weight. All rings are assumed to be commutative rings with 1, and all ring HMs are assumed to preserve 1.

- (1) State and prove the Sylow theorem(s).
- (2) Define the terms *algebraic extension*, *separable extension*, *normal extension*, *splitting field*, *Galois extension*. Prove that if $[F : E]$ is finite then F is a normal extension of E if and only if F is a splitting field for some $f \in E[x]$. You may assume the result that splitting fields are unique, but must state it carefully and correctly for full credit.
- (3) Define the Dedekind ring