

## ALGEBRA BASIC EXAM: JANUARY 2019

Attempt four of the following six questions. All questions carry equal weight. All rings are assumed to be commutative rings with 1, and all ring HMs are assumed to preserve 1.

- (1) State the Sylow theorem(s). Prove that if  $G$  is a finite group,  $H \leq G$  and  $P$  is a Sylow  $p$ -subgroup of  $G$ , then there exists  $g$  such that  $H \cap P^g$  is a Sylow  $p$ -subgroup of  $H$ .