4. Let $(X, \mathcal{M}, \mu)$ be a measurable space and let $\{f_n\}_n$ be a sequence of measurable functions $f_n : X \to [0, \infty)$ such that for every $\varepsilon > 0$ there exist $t_\varepsilon > 0$ and $E_\varepsilon \in \mathcal{M}$ with $\mu(E_\varepsilon) < \infty$ such that

$$\int_{\{f_n \ge t_\varepsilon\}} f_n \, d\mu + \int_{X \setminus E_\varepsilon} f_n \, d\mu \le \varepsilon$$

for all $n \in \mathbb{N}$. Is the inequality

$$\limsup_{n \to \infty} \int_X f_n \, d\mu \le \int_X \limsup_{n \to \infty} f_n \, d\mu$$

true? Prove or give a counter example.

5. Let

$$f(x) = \frac{e^{-3x} - e^{-4x}}{x}; \quad x > 0.$$

(i) Prove that f is Lebesgue integrable and compute $\int_0^\infty f(x)\, dx$.

(ii) Compute $\int_0^1 \frac{t^3 - t^4}{\log t}\, dt$.