Basic Examination: Measure and Integration   January 2022

- This test is **closed book**: no notes or other aids are permitted.
- The exam has a total of 5 questions and 100 points (20 each).
- You have 3 hours. You must scan and upload your solutions to the Box folder within 15 minutes afterwards.
- You may use without proof *standard* results from the syllabus (not homework) which are independent of the question asked, unless explicitly instructed otherwise. You must, however, **clearly** state the result you are using.

Below, if not stated explicitly, $(X; F;\ )$ is a measure space, $L_p = L_p(X; F;\ )$ is the standard L_p space ($p \in [1; \infty]$) and m is Lebesgue measure.

1. Let $f \in L^1(\mathbb{R})$. Prove that $F(x) := \sum_{n=-\infty}^{\infty} f(n+x)$ converges absolutely, for m-a.e. $x \in [0;1]$.

2. Let $p \in [1; \infty)$. Suppose $f_n \in L$