Department of Mathematical Sciences
Carnegie Mellon University

Basic Examination
Probability
Fall 2015

Time allowed: 120 minutes.

1. Obtain the proofs of the 1st and 2nd Borel-Cantelli lemmas from the convergence theorems of sums of independent random variables.
2. Let (Y_n) be IID RVs ta]ed69552 fvalues 1 and -1 with equal probabilities. W$_n$ll$\sum Y_n/2^n$ converge almost surely? If yes, than compute the distribution of the limit?
3. Let (X_n) be independent nonnegative RVs in L_1. We know that

$$\lim_{n \to \infty} \prod_{k=1}^{n} \mathrm{E}[X_k] = a < 1 :$$

   Will the sequence $\prod_{k=1}^{n} X_k$, $n \geq 1$, converge in distribution?

4. Let $(X_n$