Department of Mathematical Sciences
Carnegie Mellon University

Basic Examination
Probability
Spring 2018

Time allowed: 180 minutes.

1. Recite precisely the following definitions/facts/theorems/lemmas:
   (a) Tail σ-algebra. Kolmogorov's $0-1$ law.
   (b) Kolmogorov's three-series theorem on convergence of sums of IRVs.
   (c) Doob's maximal L_p inequalities for martingales.
   (d) Method of characteristic functions in weak convergence.

2. Let (X_n) be IID RVs with uniform distribution on $[0;1]$. For each of the items below describe all sequence of real numbers (a_n) such that $\sum_n a_n X_n$ converges (i) almost surely, (ii) weakly, (iii) in L_1, (iv) in L_2.

3. Let $(M_n$

6. Let (X_n) be sequence of RVs such that $X_n \to X$ (a.s) and $|X_n| \le Y \in L_1$. Let (F_n) be a filtration. Show that $E(X_n|F_n) \to E(X|F_\infty)$ (a.s) and in L_1.

7. Let (X_n) be a random walk on integers starting at 0 with the probability $0 < p < 1$ to go up and the probability