Basic Exam in Set Theory
September 3, 2019

Note: You may request elaborations on notation but not hints.

Problem 1 (10 points)

Let be an uncountable regular cardinal. Let

$$hA \quad j \quad < \quad i$$

and

$$hB \quad j \quad < \quad i$$

be two sequences of subsets of such that

$$fA \quad j \quad < \quad g = fB \quad j \quad < \quad g:$$

Prove there exists a set C that is closed and unbounded in and

$$C \setminus 4_{<} \; A \; = C \setminus 4_{<} \; B \; :$$

Reminder about notation: 4 is the diagonal intersection operator.

Problem 2 (20 points)

Let be an uncountable cardinal. Prove the following are equivalent.

(1) is a strongly inaccessible cardinal.

(2) For every $0 < \quad <$ and sequence $hA \quad j \quad < \quad i$ of subsets of ,
there exists $hB \quad j \quad < \quad i$ such that

Problem 3 (40 points)

Assume $V = L$. Let be an in nite cardinal and $= {}^{+}$. For each ordinal such that $< \; <$, let $h(\;)$ be the least $>$ such that

$$L \models \text{ZFC}$$

Problem 4 (10 points)

Let M be a transitive class model of ZFC and T be a tree on $!$ such that T 2 M. Prove that at least one of the following holds.

(1) $[T]$ M.

(2) There is a perfect subtree S of T such that S 2 M.

Additional instructions for Problem 4: You may use machinery from the proof the Cantor Perfect Set Theorem but you must explain your notation.

Your solution must be su ciently attentive to the di erence between truth in V and truth in M. If you are claiming a statement is absolute, then you need to be precise about which statement is absolute and why it is absolute, citing results from 21-602 when appropriate.

Problem 5 (10 points)

Let M be a transitive class model of ZFC. Let A and B be two structures of the same nite language, both of which belong to M. Assume that

M j= The universe of A is countable.

Suppose that thee uni -3421.9552 Tf 2f